An origin of light and electrons

– a unification of gauge interaction and Fermi statistics

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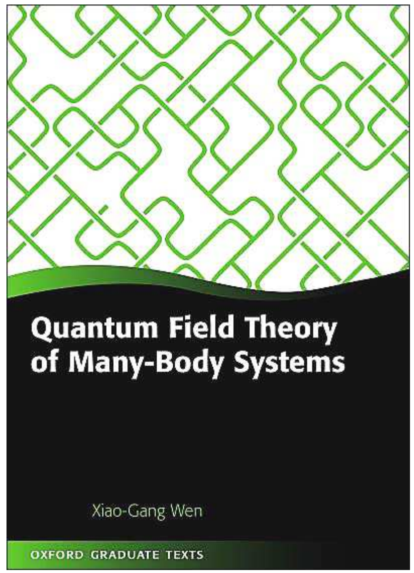
http://dao.mit.edu/˜wen

- Artificial light and quantum orders ...
  PRB **68** 115413 (2003)
- Fermions, strings, and gauge fields ...
  PRB **67** 245316 (2003)
- Strings-net condensation ...
  PRB **71** 045110 (2005)
- *Quantum field theory of many-body systems*
  (Oxford Univ. Press, 2004)



Deep mysteries of nature

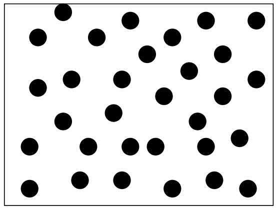
- Identical particles (Why two hydrogen atoms are exactly the same?)
- Gauge interactions (long range, massless gauge bosons)
- Fermi Statistics (Who ordered it?)
- Massless fermions (nearly, $M_f/M_P \sim 10^{-20}$)
- Chiral fermions (Are we edge excitations?)
- Gravity (The correct physical theory allows only integers)

A great-grand unification:
a single structure that explains all the mysteries

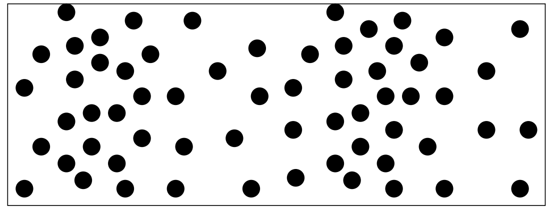
We will discuss a baby-grand unification that explains the first four mysteries from a single structure – local bosonic model.

Where do Maxwell equation and Dirac equation come from?

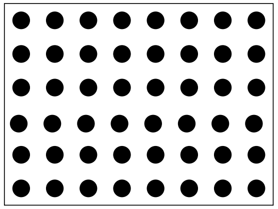
- Eular equation: $\partial_t^2 \rho - v^2 \partial_i^2 \rho = 0$ → massless scalar identical bosons



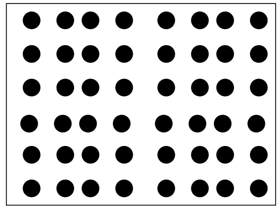
superfluid → density fluctuations

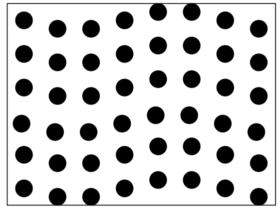


- Navier equation: $\partial_t^2 u^i - T_m^{ijk} \partial_j \partial_k u^m = 0$ → phonons (identical bosons)



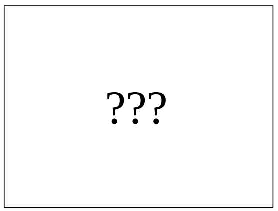
crystal → lattice fluctuations





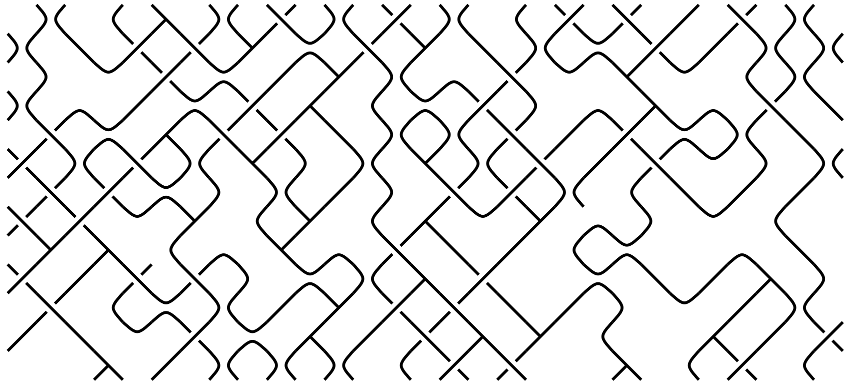
Identical particles → vacuum is not empty

- Maxwell equation: $\partial \times \mathbf{E} + \partial_t \mathbf{B} = \partial \times \mathbf{B} - \partial_t \mathbf{E} = 0$ → photons



- Dirac equation: $(\gamma^\mu \partial_\mu - m)\psi = 0$ → fermions

- Both Maxwell equation and Dirac equation can come from local bosonic models or lattice spin models if bosons/spin (a) form Long strings and (b) strings from a quantum liquid (string-net condensed state):

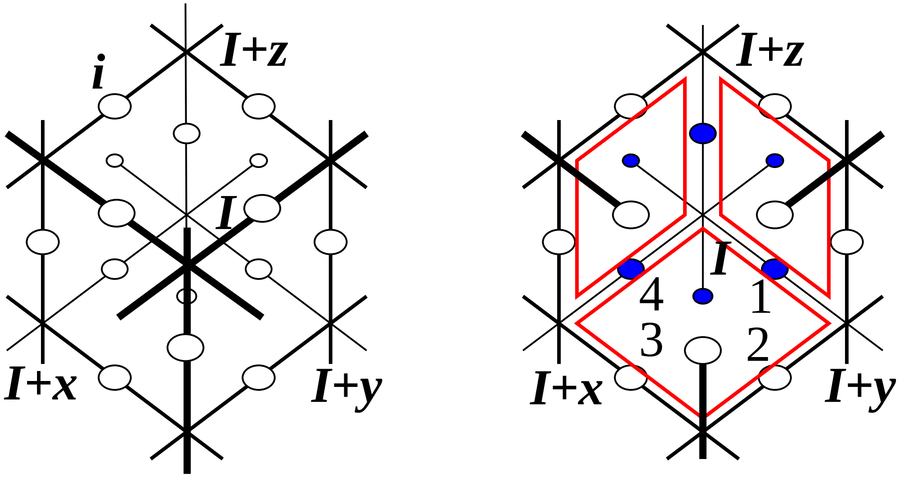


Gauge bosons and fermions can emerge as low energy collective modes of the condensed string-nets

String-net condensation provides a way to unify gauge interactions and Fermi statistics

The appearance of the gauge interaction and Fermi statistics in our nature is not an accident.

A local bosonic model on cubic lattice



A rotor θ_i on every link of the cubic lattice:

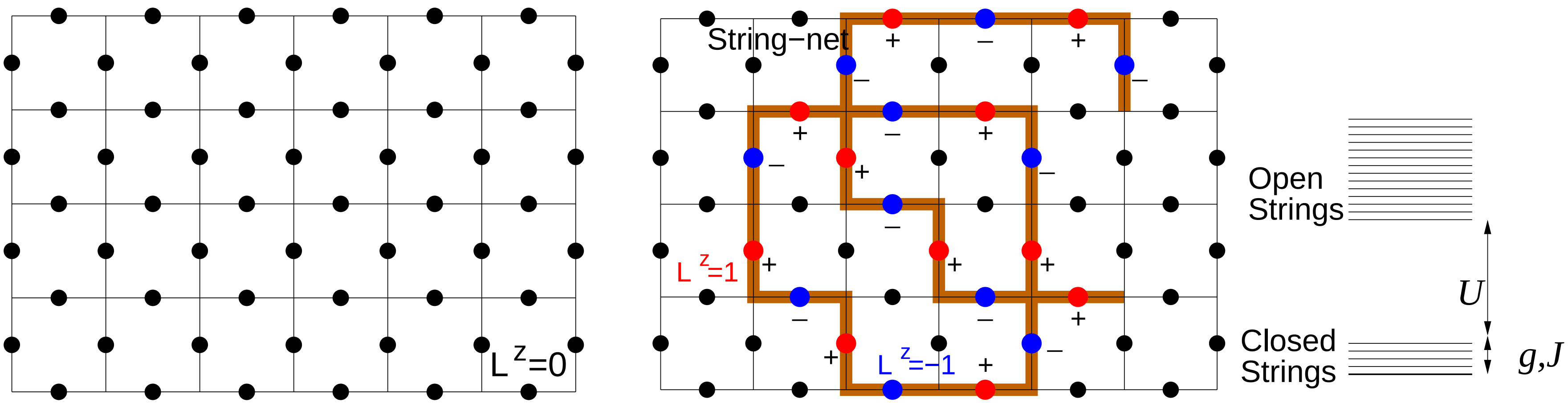
$$H = U\sum_{\mathbf{I}} Q_{\mathbf{I}}^2 - g\sum_{\mathbf{p}}(B_{\mathbf{p}} + h.c.) + J\sum_{\mathbf{i}}(L_{\mathbf{i}}^z)^2$$

$$Q_{\mathbf{I}} = \sum_{\mathbf{i}\text{ next to }\mathbf{I}} L_{\mathbf{i}}^z, \qquad B_{\mathbf{p}} = L_1^+ L_2^- L_3^+ L_4^-$$

$L^z = i\partial_\theta$: the angular momentum of the rotor

$L^\pm = e^{\pm i\theta}$: the raising/lowering operators of L^z

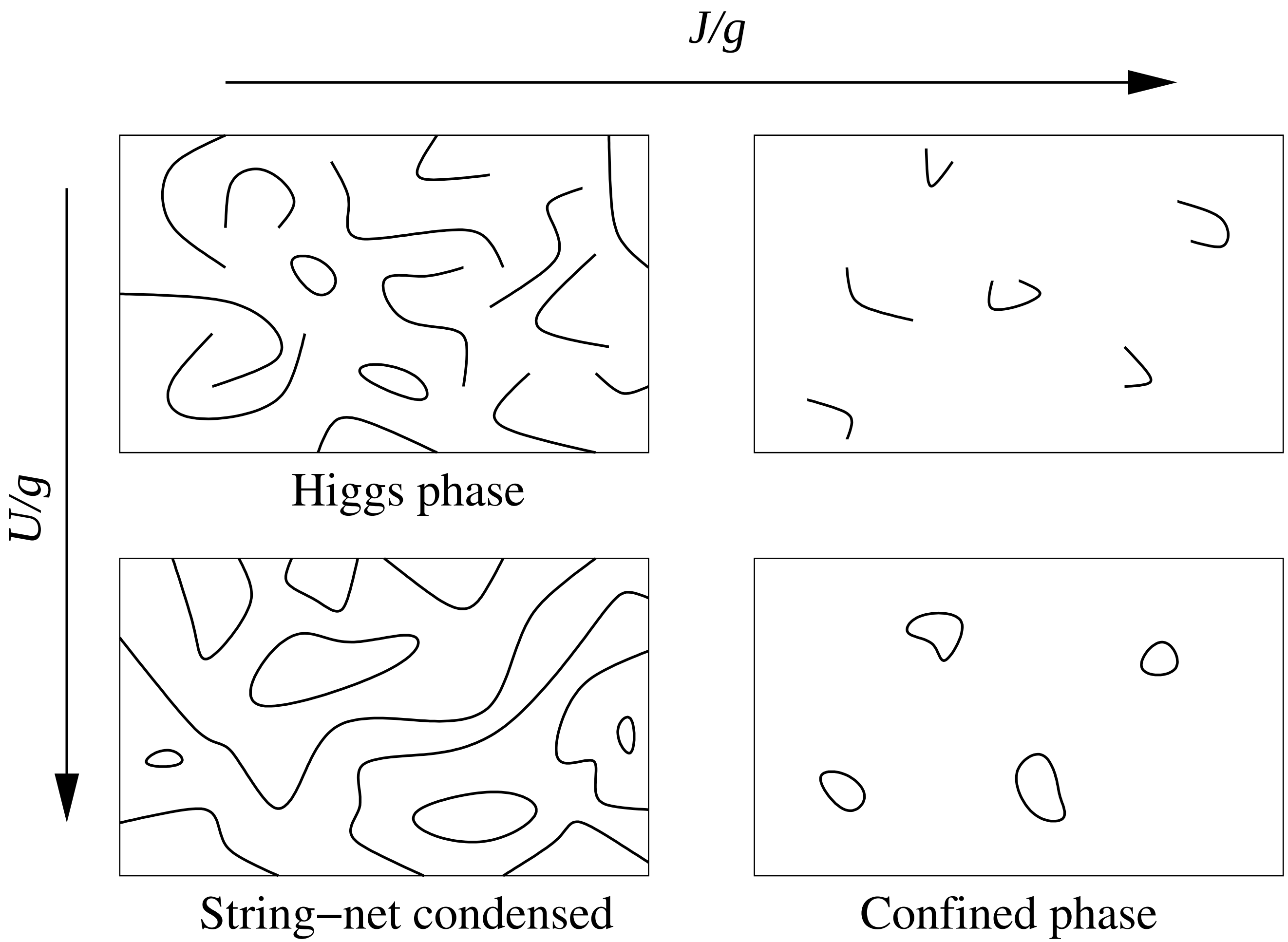
What is string-net



Physical meaning of the three terms:

- the U-term $\rightarrow$ closed strings. Open ends cost energy.
- J-term $\rightarrow$ string tension
- the g-term $\rightarrow$ strings can fluctuate

What is string-net condensation

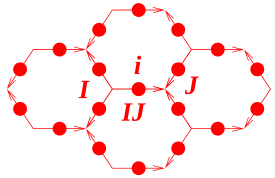


$$|\text{string-net condensed}\rangle = \sum_{\text{all closed-string-nets}} |\text{string-net}\rangle$$

Fluctuations of condensed string-nets = $U(1)$ gauge bosons

- When $U = \infty$, the rotor model can be mapped to $U(1)$ lattice gauge model, with $\theta_{\mathbf{i}}$ on link $\mathbf{IJ}$ as the $U(1)$ gauge potential:

$$\theta_{\mathbf{i}} = a_{\mathbf{IJ}},$$



- For finite U and with other perturbations:

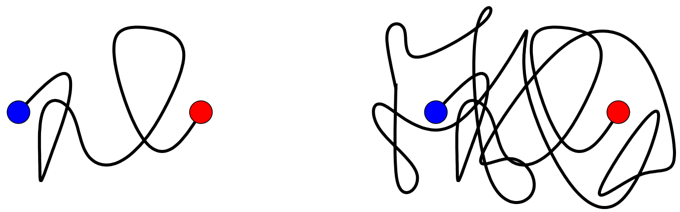
$$L_{\text{eff}} = L_{U(1)}(a_{\mathbf{IJ}}) + \epsilon_1 a_0^2 + \epsilon_2 \cos(a_{\mathbf{IJ}})$$

Since $a_{\mathbf{IJ}} = \theta_{\mathbf{i}}$ is compact, the ϵ-terms do not generate a mass for the $U(1)$ gauge bosons, if $\epsilon_{1,2}$ are small enough.

Gauge bosons and "gauge symmetry" can be emergent

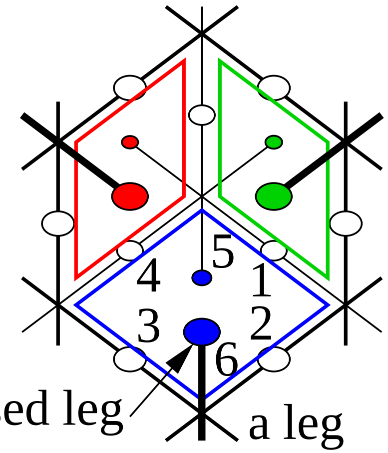
Ends of open strings = gauge charges

- Strings are unobservable in string condensed state.
- Ends of strings behave like independent particles.



- $L^z_{\mathbf{i}} \sim \dot{\theta}_{\mathbf{i}} = \dot{a}_{\mathbf{IJ}}$ correspond to electric field/flux.
  Ends of condensed strings are gauge charges
  They also carry fractional rotor-angular momentum ($L^z = \pm 1/2$)

Can get fermions for free (almost) Levin & Wen 04



Just add some legs

- Dressed-string model:

$$H = U \sum_{\mathbf{I}} Q_{\mathbf{I}} - g \sum_{\mathbf{p}} (B_{\mathbf{p}} + h.c.) + J \sum_{\mathbf{i}} (L_{\mathbf{i}}^z)^2$$

$$B_{\mathbf{p}} = L_1^+ L_2^- L_3^+ L_4^- (-1)^{L_5^z + L_6^z}$$

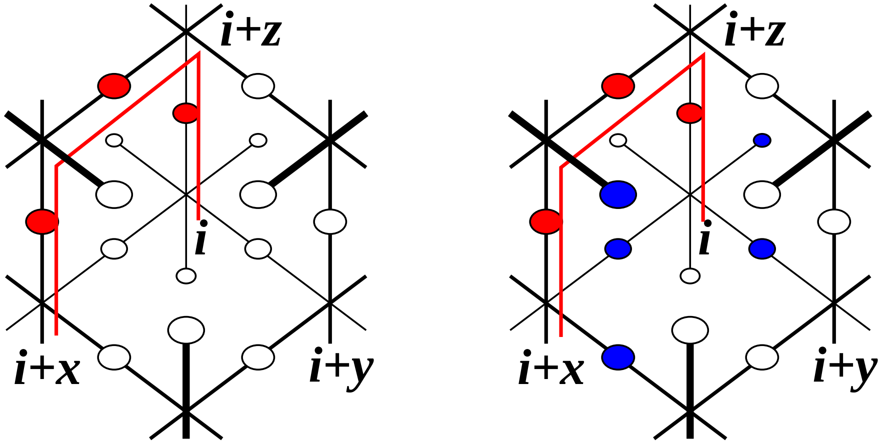
- Different ground state wave function for the string-net condensed state

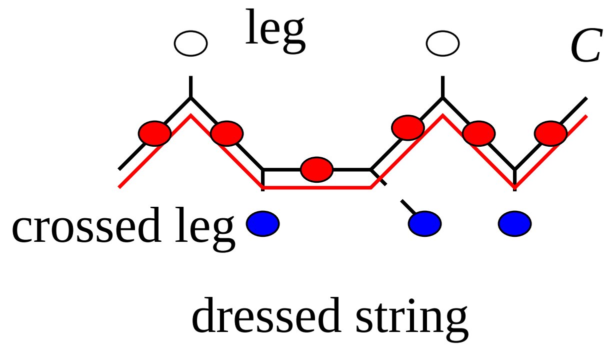
$$|\text{string-net condensed}\rangle = \sum_{\text{all closed-string-nets}} \pm |\text{string-net}\rangle$$

which leads to different statistics for the ends of condensed strings.

String operators – creation operators of gauge charges

- A pair of gauge charges is created by an open string operator which commute with the Hamiltonian except at its two ends.
  Strings cost no energy and is unobservable.





- In simple-string model – simple-string operator

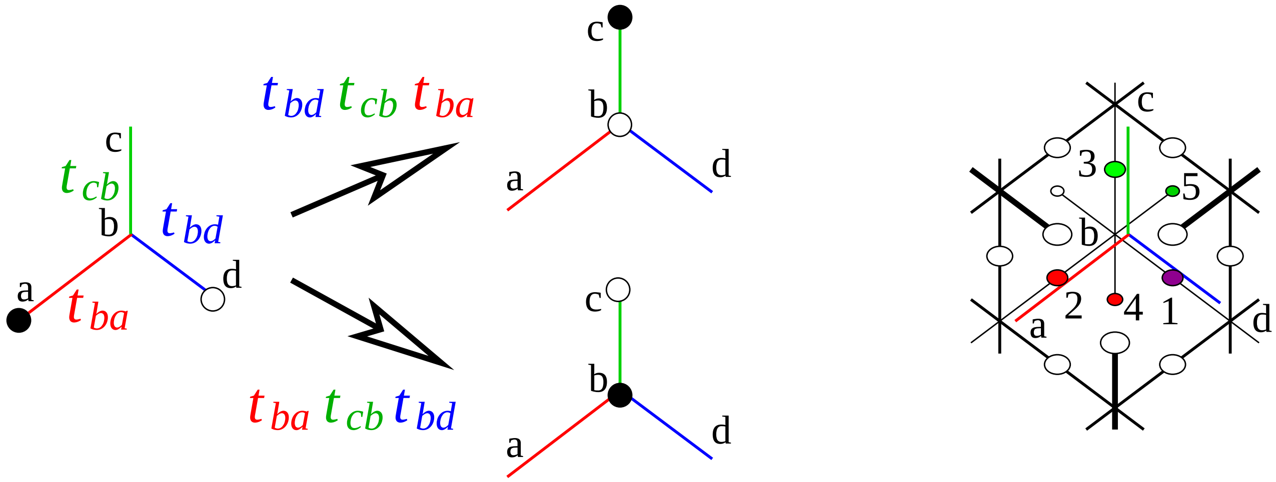
$$L^{+}_{i_1} L^{-}_{i_2} L^{+}_{i_3} L^{-}_{i_4} \ldots$$

- In dressed-string model – dressed-string operator

$$(L^{+}_{i_1} L^{-}_{i_2} L^{+}_{i_3} L^{-}_{i_4} \ldots) \prod_{i \text{ on crossed legs of } C} (-1)^{L^z_i}$$

A dressed-string operator creates a pair of fermions

- The statistics is determined by particle hopping operators Levin & Wen 02:



- An open string operator is a hopping operator of the gauge charges. Open string operator determine the statistics.
- For simple-string model: $\hat{t}_{ba} = L_2^+$, $\hat{t}_{cb} = L_3^-$, $\hat{t}_{bd} = L_1^+$
  We find $\hat{t}_{bd}\hat{t}_{cb}\hat{t}_{ba} = \hat{t}_{ba}\hat{t}_{cb}\hat{t}_{bd}$
  The ends of simple-string are bosons.
- For dressed-string model: $\hat{t}_{ba} = (-)^{L_4^z + L_1^z} L_2^+$, $\hat{t}_{cb} = (-)^{L_5^z} L_3^-$, $\hat{t}_{bd} = L_1^+$
  We find $\hat{t}_{bd}\hat{t}_{cb}\hat{t}_{ba} = -\hat{t}_{ba}\hat{t}_{cb}\hat{t}_{bd}$
  The ends of dressed-string are fermions.

What make fermions massless?

- Consider the hopping Hamiltonian for a *single* end of string

$$\hat{H} = \sum_{\mathbf{ij}} (\hat{t}_{\mathbf{ij}} + h.c.)$$

$\hat{H}$ may realize translation symmetry only projectively.

- The translation $\hat{T}_{\mathbf{a}}^{(2)}$ of the *two* ends of a string satisfies the translation algebra

$$\hat{T}_{\mathbf{a}}^{(2)} \hat{T}_{\mathbf{b}}^{(2)} = \hat{T}_{\mathbf{b}}^{(2)} \hat{T}_{\mathbf{a}}^{(2)}, \qquad \mathbf{a}, \mathbf{b} = \mathbf{x}, \mathbf{y}, \mathbf{z}$$

The translation $\hat{T}_{\mathbf{a}}$ of the *one* ends of a string satisfies

$$\hat{T}_{\mathbf{a}} \hat{T}_{\mathbf{b}} = \eta \hat{T}_{\mathbf{b}} \hat{T}_{\mathbf{a}}, \qquad \eta = \pm 1$$

- $\eta = -1 \to \pi$-flux through each square $\to$ massless fermions
- The string-net wave function $\Phi(X) = (-1)^{N_X}$ given rise to the π-flux, where N_X=number of spares enclosed by the closed string X.

Comparison with superstring theory

Superstring theory

- gauge boson = small open string of size l_P.
- fermion comes from "super world sheet" $(\sigma^1, \sigma^2, \theta^\alpha)$.
- graviton = small closed string of size l_P.

Fermions do not have to carry gauge charges.

String-net theory

Every thing comes from local bosonic model — **locality principle**

1. $\mathcal{H}_{tot} = \mathcal{H}_i \otimes \mathcal{H}_j \otimes ....$
2. Local operator = operators acting within $\mathcal{H}_i$.
3. Hamiltonian = sum of local operators.

- gauge boson = fluctuations of large string-nets that fill the space.
- fermion = one end of open string.
- graviton = ???.

Fermions (including composite fermions) must carry gauge charges.

- 123 standard model is inconsistent with the locality principle.
- $SU(5)$ GUT is inconsistent with the locality principle.
  But can be fixed by including additional discrete (say Z_2) gauge theory.
  Prediction, cosmic string associated with the discrete gauge theory.
- $SO(10)$ GUT can be consistent with the locality principle.

Summary

- Gauge interaction and Fermi statistics are just phenomena of quantum interference in infinity dimension – many-body quantum entanglements.

- No need to introduce gauge bosons and fermions by hand. They just emerge if our vacuum has a string-net condensation.

- Constructed spin model on cubic lattice that reproduce QED and QCD Wen 03.
  They are the $U(1)$ and the $SU(3)$ in the $U(1) \times SU(2) \times SU(3)$ standard model.
  But ... have trouble to get the chiral coupling of the $SU(2)$.

Six fascinating properties of nature:

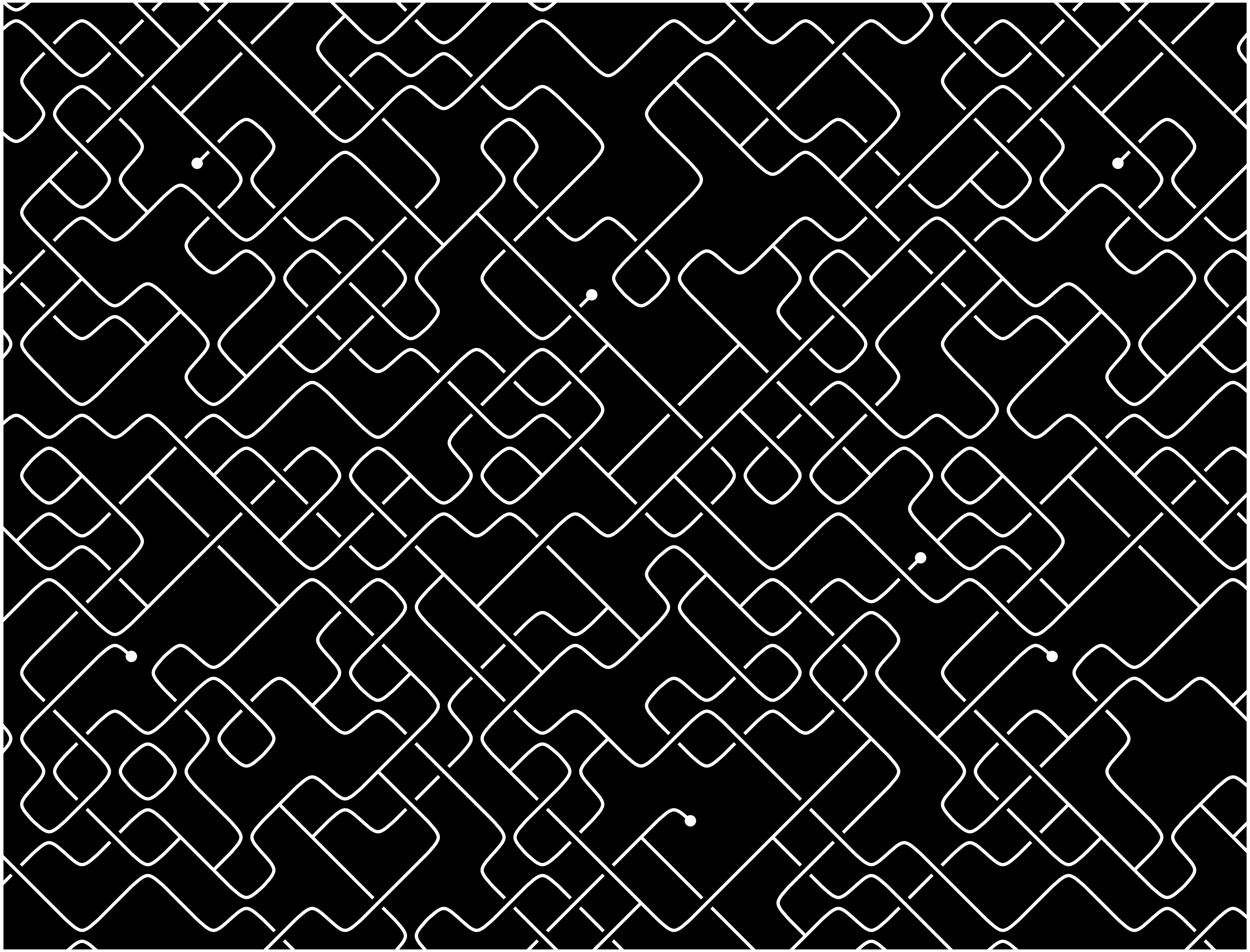
- Identical particles
- Fermi statistics
- Chiral fermions
- Gauge interaction
- Massless fermions
- Gravity

The string-net condensation picture can explain four of them.

Four down and two more to go!

A picture of our vacuum

- a recipe for making an artificial vacuum in condensed matter



General string-net condensed wave functions Levin & Wen 04

Too hard to describe $\Phi(X) \neq$ const. directly.

Indirect description:

- Types of strings: $1, 2, ..., N$. 0 represents no string.
- Branching rule: Kogut & Susskind 75

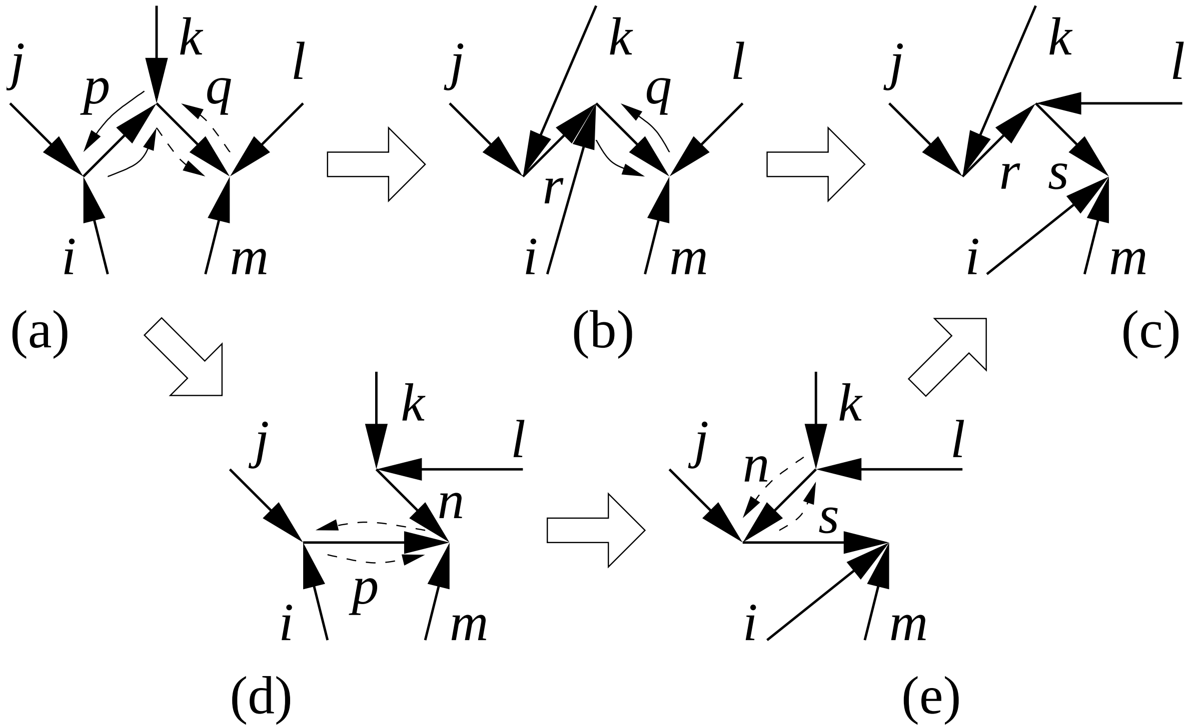
  $\delta_{ijk} = 1 \rightarrow (ijk)$ branching is allowed in ground state.

  $\delta_{ijk} = 0 \rightarrow (ijk)$ branching is not allowed in ground state.
- Topological: $\Phi(X) = \Phi(X')$ if two string-nets X and X' has the same topology. Freedman etal 03
- Rebranching relation and 6j-symbol:

Topological string-net condensation is described by a set of data $(N, \delta_{ijk}, F^{ijm}_{kln})$

- Not all sets $(N, \delta_{ijk}, F^{ijm}_{kln})$ describe consistent string-net condensation.

  Moore & Seiberg 89



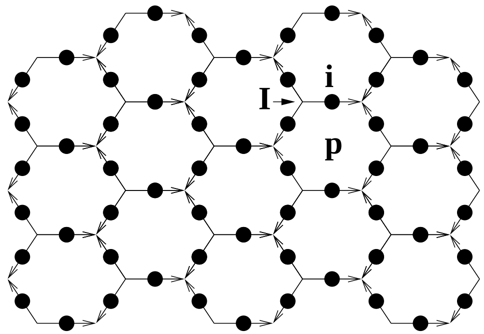
- Pentagon identity

$$\sum_n F^{mlq}_{kp^*n} F^{jip}_{mns^*} F^{js^*n}_{lkr^*} = F^{jip}_{q^*kr^*} F^{riq^*}_{mls^*}$$

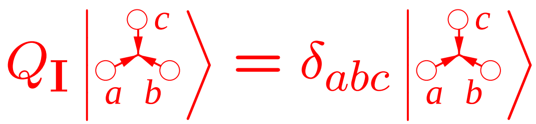
- The solutions of the above non-linear equations (called tensor categories) describe all the string-net condensed state.

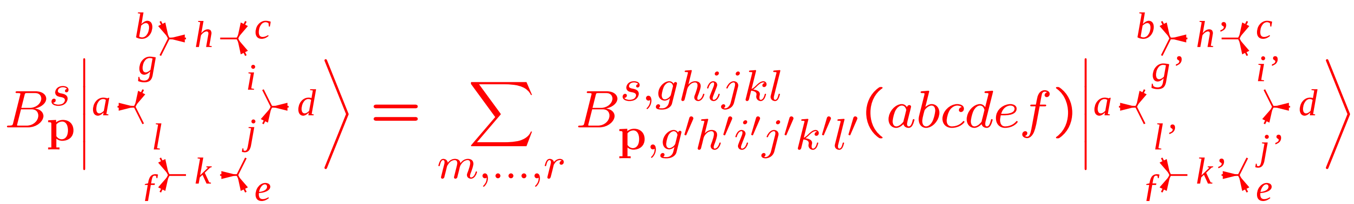
- All string-net condensed states characterized by $(N, \delta_{ijk}, F^{ijm}_{kln})$ can be realized by exactly soluble lattice models with 12 spin interactions.
- The low energy effective theories are topological theories, and almost all the topological theories can be realized this way.
- The 6-j symbol of a group G, satisfy the pentagon identity. The fluctuations of the corresponding string-net condensation $\to$ gauge boson with gauge group G.
- The 6-j symbol of a quantum group $\hat{G}$, satisfy the pentagon identity. The corresponding string-net condensation $\to$ doubled Chern-Simons theory.

- Spins on Kagome lattice: $L_{\mathbf{i}} = 0, 1, 2, \cdots$
- No string state $= |L_{\mathbf{i}} = 0\rangle$. Type-$s$ string: string of $L_{\mathbf{i}} = s$ spins
- Exactly soluble Hamiltonian is obtained from the data $(N, \delta_{ijk}, F^{ijm}_{kln})$



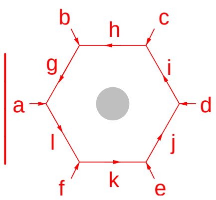
$$H_{\text{strnet}} = g\sum_{\mathbf{p}}(1 - B_{\mathbf{p}}) + U\sum_{\mathbf{I}}(1 - Q_{\mathbf{I}}), \qquad B_{\mathbf{p}} = \sum_{s=0}^{N} a_s B^s_{\mathbf{p}}$$

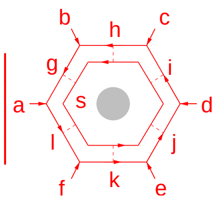




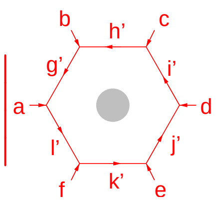
$$B^{s,ghijkl}_{\mathbf{p},g'h'i'j'k'l'}(abcdef) = F^{bg^*h}_{s^*h'g'^*} F^{ch^*i}_{s^*i'h'^*} F^{di^*j}_{s^*j'i'^*} F^{ej^*k}_{s^*k'j'^*} F^{fk^*l}_{s^*l'k'^*} F^{al^*g}_{s^*g'l'^*}$$

- $B^s_{\mathbf{p}}$ create a small loop of type-s string around hexagon $\mathbf{p}$

$B^s_{\mathbf{p}} \Big|$ 

$\Big\rangle = \Big|$ 

$\Big\rangle$

$= \sum_{g'h'i'j'k'l'} F^{bg^*h}_{s^*h'g'^*} F^{ch^*i}_{s^*i'h'^*} F^{di^*j}_{s^*j'i'^*} F^{ej^*k}_{s^*k'j'^*} F^{fk^*l}_{s^*l'k'^*} F^{al^*g}_{s^*g'l'^*} \Big|$ 

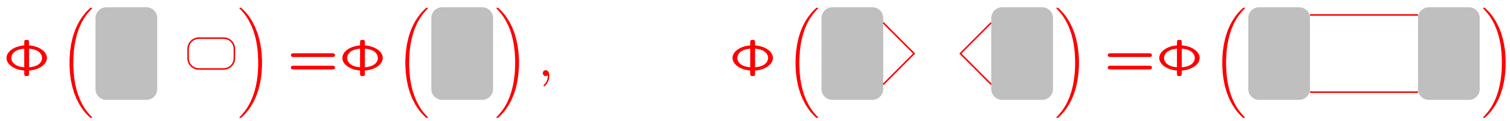
$\Big\rangle$

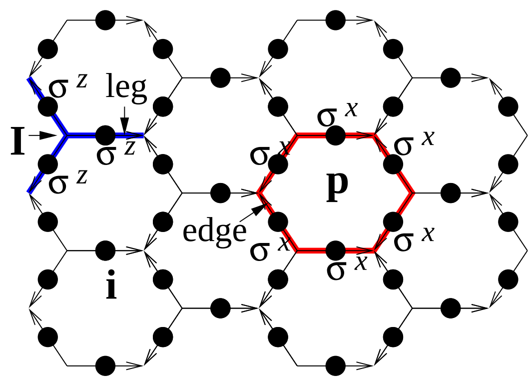
- $(\sum_s a^s B^s_{\mathbf{p}}, Q_{\mathbf{I}})$ is a commuting set of operators. H_{strnet} is exactly soluble.
- $B^s_{\mathbf{p}}$ term generates string hopping. $Q_{\mathbf{I}}$ term enforce the branching rule in ground state.

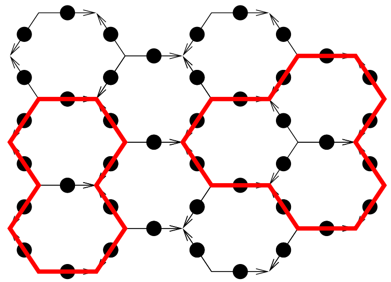
Some examples – from the solutions of the pentagon identity

Z_2 gauge theory

- $N = 1$, $\delta_{000} = \delta_{110} = 1, \delta_{100} = 0$ (only closed strings), F^{ijm}_{kln} leads to







- The Hamiltonian

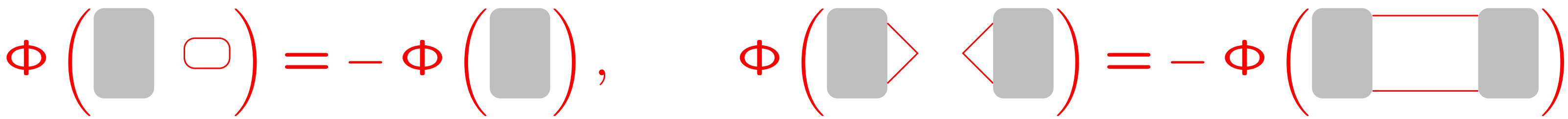
$$H_{\text{strnet}} = g \sum_{\mathbf{p}} \prod_{\text{edges of } \mathbf{p}} \sigma^x_{\mathbf{i}} + U \sum_{\mathbf{I}} \prod_{\text{legs of } \mathbf{I}} \sigma^z_{\mathbf{i}}$$

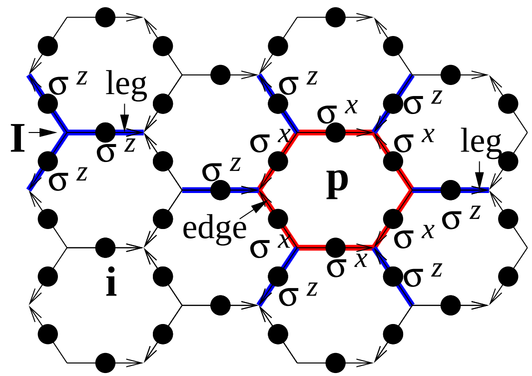
- Ground state wave function $\Phi(X) = \text{const.}$
- Effective theory: Z_2 gauge theory $= U(1) \times U(1)$ Chern-Simons theory

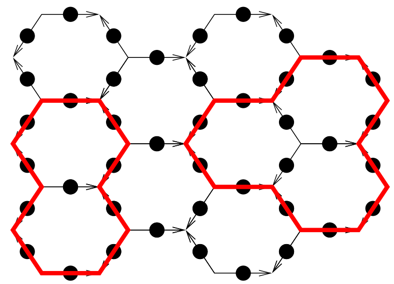
$$L = \frac{1}{4\pi} K_{IJ} a_{I\mu} \partial_\nu a_{J\lambda} \epsilon^{\mu\nu\lambda}, \qquad K = \begin{pmatrix} 0 & 2 \\ 2 & 0 \end{pmatrix}$$

Doubled semion theory

- $N = 1$, $\delta_{000} = \delta_{110} = 1, \delta_{100} = 0$ (only closed strings), F^{ijm}_{kln} leads to







- The Hamiltonian

$$H_{\text{strnet}} = -\sum_{\mathbf{I}} \prod_{\text{legs of } \mathbf{I}} \sigma^z_{\mathbf{i}} + \sum_{\mathbf{p}} \Big(\prod_{\text{edges of } \mathbf{p}} \sigma^x_{\mathbf{j}} \Big) \Big(\prod_{\text{legs of } \mathbf{p}} (-)^{\frac{1-\sigma^z_{\mathbf{j}}}{4}} \Big)$$

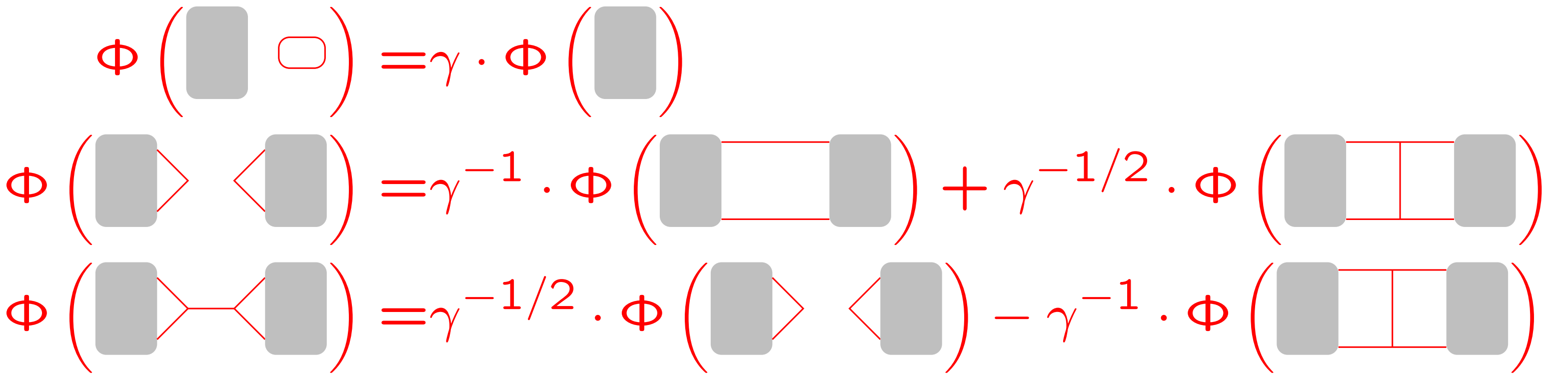
- Ground state wave function $\Phi(X) = (-)^{X_c}$, where X_c is the number of loops in the string configuration X
- Effective theory: $U(1) \times U(1)$ Chern-Simons theory

$$L = \frac{1}{4\pi} K_{IJ} a_{I\mu} \partial_\nu a_{J\lambda} \epsilon^{\mu\nu\lambda}, \qquad K = \begin{pmatrix} 2 & 0 \\ 0 & -2 \end{pmatrix}$$

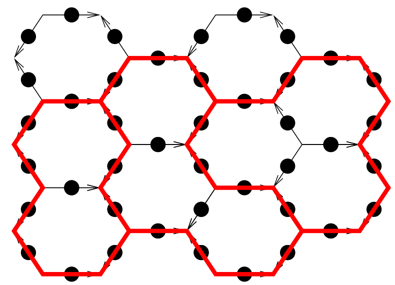
- Ends of open strings → Semions with $\theta = \pm\pi/2$

Doubled Yang-Lee theory

- $N = 1$, $\delta_{000} = \delta_{110} = \delta_{111} = 1, \delta_{100} = 0$ (branched string-nets), F^{ijm}_{kln} leads to



where $\gamma = \frac{1+\sqrt{5}}{2}$



- Ground state has a string-net condensation
- Effective theory: $SO_3(3) \times SO_3(3)$ Chern-Simons theory
- Ends of open strings $\rightarrow$ particles with non-Abelian statistics