Sideways Tunnelling and Fractional Josephson Frequency in Double Layered Quantum Hall Systems*

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ABSTRACT: The filling fraction $1/k$ double layered quantum Hall state has been shown to exhibit many superfluid properties including the appearance of an alternating tunnelling current in response to an external voltage V. Here we propose the notion of "sideways tunnelling", in which a current tunnels between two such $1/k$ states through an incompressible fractional quantum Hall state of filling fraction $2/m$. We predict an alternating tunnelling current or a narrow band noise at a frequency $\omega = \frac{eV}{\hbar m}$, as a consequence of the tunnelling of e/m fractional charges. We also discuss singularities in the tunnelling current noise spectrum and the non-linear I-V curve.

* Published in *Europhys. Lett.* **35**, 227 (1996)

In Ref. 1 we predicted within the semiclassical approximation and in the weak tunnelling limit, that a DC voltage V applied across a double-layered quantum Hall system[2] in certain special states (to be characterized below) would produce a tunnelling current alternating with frequency $\omega = eV/\hbar$. While our effect is reminiscent of the Josephson effect[3] there are some important differences. Here the frequency is half of the Josephson frequency: the charge carrier is not a pair of electrons as in a superconductor.[4] We claimed that there is a superfluid lurking in certain states of double-layered quantum Hall systems. In this paper, we propose and discuss a novel notion of "sideways tunnelling" involving the juxtaposition of three double layered quantum Hall systems. We will neglect interlayer tunnelling[5] in the main part of this paper.

In the effective Chern-Simons gauge field theory[6] description of quantum Hall fluids, a double layered system is described by

$$\mathcal{L} = \frac{1}{4\pi}(\sum_{I,J} \varepsilon_{\mu\nu\lambda} a_{I\mu} K_{IJ} \partial_\nu a_{J\lambda} + 2\sum_{I} \varepsilon_{\mu\nu\lambda} A_\mu \partial_\nu a_{I\lambda}) + \text{Maxwellterms} \tag{1}$$

Here $I, J = 1, 2$ and K is a 2×2 matrix. The electromagnetic current in layer I is given by $J_\mu^{(I)} = \frac{1}{2\pi}\epsilon^{\mu\nu\lambda}\partial_\nu a_{I\lambda}$ with the total density and current $J_\mu = J_\mu^{(1)} + J_\mu^{(2)}$ coupling to the electromagnetic potential A_μ. The difference $J_\mu^{(-)} = J_\mu^{(1)} - J_\mu^{(2)}$ describes the relative density and current fluctuations in the two layers. When the matrix K has a zero eigenvalue, the corresponding eigenmode describes a superfluid. For the sake of simplicity and definiteness we specialize to the matrices $K = \begin{pmatrix} k & k \\ k & k \end{pmatrix}$ with k an odd integer. (This describes the class of states studied in Ref. 1 and corresponds to the wave function (kkk) in Halperin's notation[7].) The mode $a_- = a_1 - a_2$ is then governed by Maxwell dynamics (which is gapless[8]), rather than Chern-Simons dynamics, and thus describes a superfluid.

In our formalism, interlayer tunnelling events correspond to the presence of magnetic monopoles in space-time. When an electron tunnels from one layer to the other

$$\pm 2 = \int d^2x dt\ \partial^\mu J_\mu^{(-)} = \int d^2x dt\ \frac{1}{2\pi}\partial^\mu \epsilon_{\mu\nu\lambda}\partial_\nu \alpha_-^\lambda \tag{2}$$

The discrete character of the electron implies Dirac quantization of the magnetic monopole associated with the gauge potential a_- in Eculidean 3-space. Going though the analysis of Ref. 1, we found that an applied voltage produces an alternating current of frequency $\omega = eV/\hbar$. Note that the frequency does not depend on k. This is because only electron can tunnel from one layer to the other, since they are separated by vacuum. Thus the frequency of the interlayer tunnelling does not reflect the fractional charge of the quasiparticles.

To have more precise description of tunnelling, let us consider the "noise" spectrum defined by

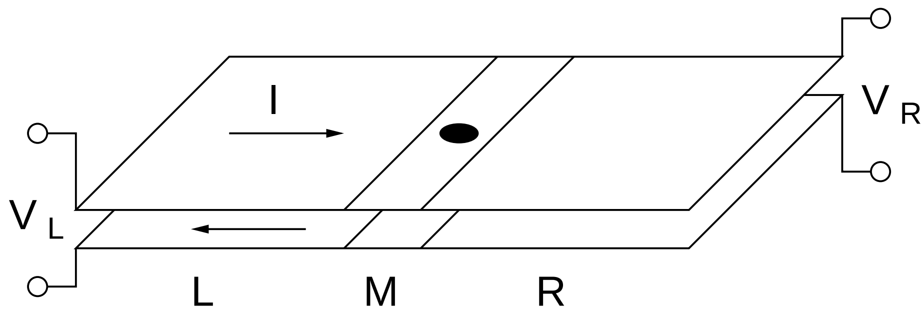
$$F(\omega) = \int_0^\infty dt\ \langle j_T(t) j_T(0)\rangle e^{-i\omega t} \tag{3}$$

with j_T the interlayer tunnelling current

$$j_T = i\int d^2x[t(x)c_1^\dagger(x)c_2(x) - h.c.]. \tag{4}$$

(where c_i represents an electron annihilation operator in layer i.) An ideal Josephson effect is defined by a δ-function in $F(\omega)$ at the Josephson frequency. This occurs for interlayer tunnelling at zero temperature (within perturbation theory in the tunnelling amplitude t).[1]

So far we have reviewed the situation discussed in Ref. 1. Let us now come to what we call "sideways tunnelling". From now on we will assume for simplicity that there is no interlayer tunnelling.



i Fig. 1: Sideways tunnelling junction . The tunnelling currents flow in opposite directions in the first and second layers.

We imagine an experimental situation in which three double-layered systems characterized by matrices K_L, K_M, and K_R (with L, M, R for left, middle, and right respectively) are connected together as shown in figure 1. For definiteness let us take K_L and K_R to be $\begin{pmatrix} k & k \\ k & k \end{pmatrix}$ and K_M to be $\begin{pmatrix} m & 0 \\ 0 & m \end{pmatrix}$ with m an odd integer. We apply a voltage V_R between the top and bottom layers in the R region and V_L in the L region.

Consider a quasiparticle of charge e/m tunnelling sideways from the left edge to the right edge of the middle region in the top layer, and at the same time a quasihole tunnelling in the same direction in the bottom layer. The coupling between the order parameter $\langle c_1^\dagger c_2 \rangle$ in the R region and the excitations on the right edge of the M region is induced by an electron tunnelling across the boundary in the top layer and a hole tunnelling in the bottom layer. A similar coupling exists at the boundary between the L and M region. This would be made more precise later. Assuming equilibrium between the bulk of the R, L regions and the two edges of the M region, we see that the voltage difference between the top and bottom layers at the right edge of the M region is given by V_R and at the left edge by V_L. Thus the quasiparticle tunnelling process described above costs an energy $e(V_R - V_L)/m$. If the tunnelling is coherent, the frequency of the alternating current or the frequency of the peak in noise spectrum will be

$$\omega = \frac{\Delta E}{\hbar} = \frac{1}{m} e|V_L - V_R|/\hbar \equiv \omega_m \tag{5}$$

In general the tunnelling noise spectrum may also have peaks at multiples of the above frequency. The peak at $\omega = m\omega_m$ is due to electron tunnelling. Since the long range orders exist in the R and L regions, this peak should be a δ-function peak (at zero temperature).

Before going into more detailed discussions, we would like to make a few remarks. If $k \neq m$, then by charge conservation, quasiparticle tunnelling necessarily involves gapless edge excitations on the boundary between the L and M regions, and of course also on the boundary between the R and M regions. We will see later that the peak at $\omega = \omega_m$ can be an algebraic singularity (due to interferences with the edge excitations), or a δ-function singularity, depending on the strength of the coupling to edge excitations.

We also like to remark that edge excitations on the two sides of M region carry different momenta. To conserve momentum in the tunnelling process, impurities must intervene: in other words, an excitation going from the left edge to the right edge must scatter off and exchange momentum with impurities. For simplicity first ignore the R and L regions. Quasiparticle tunnelling between the two edges assisted by a single impurity already contains an algebraic singularity at a fractional Josephson frequency in its noise spectrum.[9] But in general there are many impurities, impurities a, b, c, *etc.* , with effective voltages across each impurity (*i.e.,* the effective value of $V_R - V_L$ near each impurity, rather than the experimentally imposed $V_R - V_L$) $V_a \neq V_b$...... and so the algebraic singularity in the $F(\omega)$ would be completely smeared. At this point, the hidden superfluid comes to the rescue. For the K_L and K_R we chose, there is a superfluid in the left and right regions. In the superfluid the voltage difference between the two layers are constant in space (*i.e.,* V_L and V_R are constant in the region R and L). The superfluidity insures that all the effective voltages $V_a, V_b, ...$ are equal, and hence guarantees the existence of, at least, an algebraic singularity at $\omega = \omega_m$. This is the simplest example that the superfluidity in the (kkk) enhance the singularity in the tunnelling current noise spectrum at $\omega = \omega_m$.

In the following we will give a more detailed study of the sideways tunnelling. The coupling between the L region and the R region is not direct and involves three different couplings: the first coupling is induced by electron tunnelling between the edge of the R region and the right edge of the M region, the second one is induced by the quasiparticle tunnelling between the right and the left edges of the M region, and the third one is induced by electron tunnelling between the edge of the L region and the left edge of the M region. We see that the detailed description of the tunnelling is quite complicated. In the following we will only consider two limiting cases: the edges of the R region and the L region are either strongly or weakly coupled to the edges of the M region.

To gain a better understanding of sideways tunnelling and the effects of edge excitations, let us give a brief description of the relevant edge states. For our choice of $K_{L,R,M}$, there are three branches of edge excitations on the boundary between the L and M regions and on the boundary between the R and M regions. The K_R state in the R region has one branch of edge excitations. Let ψ_{eR} denote the electron operator in that edge branch. The electron creation operators at the edge of the R region in the top and bottom layers are given respectively by

$$c^\dagger_{R1} = \psi^\dagger_{eR} e^{-i\theta_R/2}, \quad c^\dagger_{R2} = \psi^\dagger_{eR} e^{i\theta_R/2} \tag{6}$$

, where θ_R is the phase of the superfluid order parameter in the R region. The K_M state contains two branches of edge excitations associated with the top and the bottom layers. Let χ_{qRi} to be the quasiparticle operator in the i^{th} layer and at the right edge of the middle region. The operators $\chi_{eRi} = (\chi_{qRi})^m$, carrying charge e, are the corresponding electron operators. Similarly we can define ψ_{eL}, c_{Lj}, χ_{qLj}, and χ_{eLj}.

Now the coupling between the R and M regions can be described by the following operator (and its hermitian conjugate)

$$A_{RM} = \int dx \; t_R (c^\dagger_{R1}\chi_{eR1} + c^\dagger_{R2}\chi_{eR2}) \tag{7}$$

induced by electron tunnelling. Because of the superfluid correlation $\langle c^\dagger_{R1} c_{R2}\rangle \propto e^{i\theta_R}$, A_{RM} induce an effective interlayer coupling between χ_{eR1} and χ_{eR2}:

$$B_{RM} = \int dx\ t'_R \chi^\dagger_{eR2} \chi_{eR1} e^{-i\theta_R} \tag{8}$$

Similarly, A_{LM} and B_{LM} are defined by substituting everywhere L for R.

The quasiparticle tunnelling between the two edges of the middle region described above is given by

$$A_M = \int dx\ t_M \chi_{qR1} \chi^\dagger_{qR2} \chi^\dagger_{qL1} \chi_{qL2} \tag{9}$$

Microscopically, the tunnelling in the first layer and the tunnelling in the second layer do not occur at the same time. But since we are only interested in low energies, with a corresponding time scale much longer than the time difference in question, we are justified in taking the tunnelling in the first layer and in the second layer to occur at the same time in this effective Hamiltonian. Note that A_M refer only to the M region, while $A_{RM,LM}$ and $B_{RM,LM}$ link the M region with the R and L regions. In general the tunnelling amplitudes t_R, t_L, t'_R, t'_L, and t_M depend on x due to impurities. The tunnelling current across the middle region is given by

$$I_-(t) = \int^t dt' (e^{i\frac{e}{m}(V_L - V_R)t'} \langle [A^\dagger_M(t'), A_M(0)]\rangle + c.c.) \tag{10}$$

(Here we have assumed $t_M \ll t_R, t_L$ and the equilibrium between the edge and the bulk.) Note I_- is the difference between the tunnelling currents in the top and the bottom layer $I_- = I_1 - I_2$. The total tunnelling current $I_1 + I_2$ vanishes for the tunnelling described by H_M.

We will now determine the long time behavior of the correlation between $A_M(t)$ and $A_M(0)$, and hence by (10) the frequency characteristic of the current I_-. We start with the long time behavior between various relevant operators known from the theory of the edge excitations[9] (a schematic notation suffices for our purpose):

$$\begin{aligned} \langle \chi^\dagger_q(t) \chi_q(0)\rangle &\sim \frac{1}{t^{1/m}} \\ \langle \chi^\dagger_e(t) \chi_e(0)\rangle &\sim \frac{1}{t^m} \\ \langle \psi^\dagger_e(t) \psi_e(0)\rangle &\sim \frac{1}{t^k} \end{aligned} \tag{11}$$

Using these known facts, we now determine the scaling dimensions of various operators. In the following we will assume that t_R, t_L and t_M have random phases and short range correlation so that $\langle t_R(x) t_R(y)\rangle \sim \delta(x-y)$ *etc.* .

After averaging (7) over the location of impurities, we induce in the action the term

$$\delta S \propto \int dt dt' dx\ c^\dagger_{R1}(x,t) \chi_{eR1}(x,t) c_{R1}(x,t') \chi^\dagger_{eR1}(x,t')$$

From (11) and (6),

$$\langle (c^\dagger_{Rj} \chi_{eRj})_{x,t} (c_{Rj} \chi^\dagger_{eRj})_0 \rangle \propto (x^2 - v^2 t^2)^{-(k+m)/2}$$

. Now we simply count powers and see easily that $\delta S \sim t^3/t^{(k+m)}$. Hence A_{RM} (and A_{LM}) are irrelevant at long time and long distance if $k+m-3>0$. Similarly, averaging B_{RM} over the locations of impurities, we induce a

$$\delta S \propto \int dt dt' dx (\chi_{eR1}\chi^\dagger_{eR2})(\chi_{eR2}\chi^\dagger_{eR1}) \sim 1/t^{2m-3}$$

schematically. Again, counting powers we see that B_{RM} and B_{LM} are irrelevant if $2m-3>0$.

Two situations need to be discussed separately. When the two above inequalities are satisfied, and if t_R, t_L are small, the correlation function of A_M can be calculated neglecting A_{RM}, A_{LM} and B_{RM}, B_{LM}. We find, averaging over t_M, that $\langle [A^\dagger_M(t), A_M(0)]\rangle \propto L_{edge} t^{-4/m}$, where L_{edge} denotes the length of the edge. Thus, in the weak coupling limit, the I-V curve has the form $I_- \propto (V_L - V_R)^{\frac{4}{m}-1}$ and the singularity in the noise spectrum $F(\omega)$ is given by

$$F(\omega) \propto \frac{1}{\Gamma(4/m)} (\omega - \omega_m)^{\frac{4}{m}-1} \tag{12}$$

In this discussion we have ignored the potential interaction between the edge branches. This potential interaction may change the above exponent, *i.e.,* one should replace $4/m$ by a new exponent γ whose value depends on the interaction.

When t_R, t_L are large and/or one of A_{RM}, A_{LM} and B_{RM}, B_{LM} are relevant, the coupling B_{RM}, B_{LM} coupling not only makes $\langle \chi^\dagger_{eR2}\chi_{eR1}\rangle \neq 0$, it also makes $\langle \chi^\dagger_{qR2}\chi_{qR1}\rangle \neq 0$, as we will show below. The latter implies the ideal Josephson effect (at $T=0$), *i.e.,* a finite I_- even when $V_R - V_L = 0$ and a δ-function singularity in $F(\omega)$ at frequency $\omega = \omega_m$ determined by the fractional charge. When $\langle \chi^\dagger_{qR2}\chi_{qR1}\rangle \neq 0$, its phase is not uniquely determined by θ_R. The system chooses one of m different ground states characterized by $\langle \chi^\dagger_{qR2}\chi_{qR1}\rangle \propto e^{i(\theta_R + 2n\pi)/m}$, where $n = 0, ..., m-1$.

To show that the coupling (8) can indeed induce a non-zero $\langle \chi^\dagger_{qR2}\chi_{qR1}\rangle$, let us consider a simple case that t'_R in (8) proportional to a δ-function. The tunnelling events induced by (8) can be described by a 1D gas (in time direction) in the path integral formalism, as explained in Ref. 1. Each tunnelling event corresponds to a particle of the gas. The long time correlation $\langle (\chi^\dagger_{eR2}\chi_{eR1})_t (\chi^\dagger_{eR1}\chi_{eR2})_0\rangle \sim \frac{1}{t^{2m}} \sim e^{-2m\log t}$ can be intepreted as logarithmic interaction $-2m\log|t_1 - t_2|$ between the particles and hence the gas is Coulomb. The properties of such 1D Coulomb gas has been studied in Ref. 10 and depend on the strength of the coupling m. The tunnelling strength t'_R corresponds to the fugacity and hence controls the density of the gas. Clearly, for stronger tunnelling, the gas would be denser.

The gas, in general, have two phases if the Coulomb attraction is strong $m > 1$: the plasma phase (if t'_R is large and hence the gas is dense) and the confining phase (if t'_R is small and hence the gas is dilute). If the Coulomb attraction is weak $m \leq 1$ the gas is always in the plasma phase. (In the present context, m cannot be less than 1 and the $m = 1$ case does not involve tunnelling of fractional charges.)

In the plasma phase positive and negative charges move freely and completely screen any added charges, that is, the added charges no longer interact through the long range logarithmic potential). Insertion of the operator $\chi^\dagger_{qR2}\chi_{qR1}$ is just like adding charges to the Coulomb gas. Thus the correlation of $\chi^\dagger_{qR2}\chi_{qR1}$ in the time direction can be written as

$$\langle(\chi^\dagger_{qR2}\chi_{qR1})_t(\chi^\dagger_{qR1}\chi_{qR2})_0\rangle \propto e^{-V(t)} \tag{13}$$

where $V(t)$ is the potential between the added charges. With V short ranged, $\chi^\dagger_{qR2}\chi_{qR1}$ has a long range correlation.

For small t'_R and $m>1$, the gas is in the confining phase with the positive and negative charges forming dipole bound states, which can only modify the strength of the logarithmic Coulomb interaction between the added charges. With V logarithmic, $\chi^\dagger_{qR2}\chi_{qR1}$ decays algebraically, thus implying an algebraic singularity in the noise spectrum $F(\omega)$.

We have shown that for small t'_R and at $T=0$, the noise spectrum $F(\omega)$ contains an algebraic singularity at $\omega=\omega_m$. At finite temperatures this algebraic singularity is expected to be smeared to have a finite width of order T. For large t_R and at $T=0$, the noise spectrum $F(\omega)$ contains a δ-function peak. At finite temperatures the δ-function singularity is smeared to, at most, an algebraic singularity. This is because the order parameters in the R,L regions have an algebraic decay at finite temperatures.

In the presence of interlayer tunnelling, all the algebraic and δ-function singularities in the sideways tunnelling discussed here are expected to be modified below an energy scale Δ proportional to the square root of the interlayer tunnelling amplitude,[8,1] an energy scale describing the gap for the relative density fluctuations between the two layers. They may be smeared into peaks with finite width Δ since the $U(1)$ symmetry is explicitly broken by the interlayer tunnelling. However, as long as Δ is small we would still expect to see narrow band noises at multiples of ω_m.

This work is supported by the National Science Foundation under Grant No. DMR91-14553 (for XGW) and PHY89-04035 (for AZ). One of us (AZ) also thanks the Ecole Normale Superieure for hospitality and support.

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