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Neutral superfluid modes and "magnetic" monopoles in multi-layer quantum Hall systems*

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ABSTRACT: We show that, in the absence of interlayer hopping, the $\nu = 1/m$ quantum Hall states in double-layer systems contain a neutral gapless mode with linear dispersion, describing the relative fluctuations of the electron densities in the two layers. At finite temperature the system experiences a Kosterlitz-Thouless transition. In the presence of interlayer hopping an energy gap proportional to the square root of the hopping amplitude will be opened. This, in field theory, corresponds to $U(1)$ gauge field acquiring a mass due to the monopole-antimonopole plasma in the $(2+1)$-dimensional spacetime.

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In the past few years, several groups studied multi-layer quantum Hall (QH) and found many interesting properties,[1,2,3] such as the collapse of the integral quantum Hall (IQH) states at odd-integer filling fractions and the appearance of $\nu = 1/2$ FQH state in double-layer systems. In this paper we are going to study the $\nu = 1$ QH state in double-layer systems. Two energy scales play very important role here. One is the potential energy V between electrons in different layers. The other is the energy gap Δ_{SAS} between the symmetric and the anti-symmetric wave functions in the double wells. Δ_{SAS} measures the electron hopping amplitude between the two layers. When Δ_{SAS} is large, the double-layer system is equivalent to a single-layer system because all electrons are in the subband of the symmetric wave function. Here we are going to study the opposite limit, *i.e.,* $V \gg \Delta_{SAS}$. We will argue that in the absence of interlayer hopping, the double-layer state (mmm) at filling fraction $1/m$ supports neutral gapless excitations with a linear dispersion relation, the Nambu-Goldstone (NG) mode from a spontaneously broken $U(1)$ symmetry. If a non-zero interlayer hopping is present, an energy gap proportional to the square root of the the hopping amplitude will be opened. The above results for a special case of the (111) QH state have been obtained in Ref. 4,5. We also discuss superfluid properties in (mmm) states and some experimental consequences.

Consider the double-layer state $\Psi_{(lmn)}$ introduced in Ref. 6:

$$\Psi_{(lmn)} = \prod (z_i - z_j)^l \prod (w_i - w_j)^m \prod (z_i - w_j)^n e^{-\frac{1}{4}(\sum |z_i|^2 + \sum |w_i|^2)} \tag{1}$$

where z_i and w_i are the electron coordinates in the two layers. This state is favored by electrons with short range repulsions. In the following we would like argue (1) supports gapless excitations when $l = m = n$. To avoid complications from the edge excitations, let us put the system on a sphere. Introducing the spinor coordinate[7] u_i, v_i for the electrons in the first layer and $\tilde{u}_i$, $\tilde{v}_i$ for the electrons in the second layer, we may write the wave function (1) as

$$\Psi_{(lmn)} = \prod_{i<j}^{N_1} (u_i v_j - v_i u_j)^l \prod_{i<j}^{N_2} (\tilde{u}_i \tilde{v}_j - \tilde{v}_i \tilde{u}_j)^m \prod_{i=1}^{N_1} \prod_{j=1}^{N_2} (u_i \tilde{v}_j - v_i \tilde{u}_j)^n \tag{2}$$

where N_1 and N_2 are the numbers of the electrons in the two layers. The total power of u_i and v_i and the total power of $\tilde{u}_i$ and $\tilde{v}_i$, for each fixed i, must be equal to the number of the flux quanta passing through the sphere, N_ϕ. Therefore N_1 and N_2 must satisfy

$$\begin{aligned} l(N_1 - 1) + nN_2 &= N_\phi \\ m(N_2 - 1) + nN_1 &= N_\phi \end{aligned} \tag{3}$$

In general the solution of (3) is unique, representing an incompressible homogeneous ground state. But when $l = m = n$ (3) does not have a unique solution. Any N_1 and N_2 that satisfy $N_1 + N_2 = N_\phi + 1$ will satisfy (3). Those states with different $N_1 - N_2$ represent the gapless excitations, because moving electrons from one layer to the other hardly changes the short distance correlations between the electrons. We expect the total energy of the states to have the following form in the thermodynamic limit

$$E = \int d^2x \frac{1}{2} \kappa (n_1 - n_2)^2 \tag{4}$$

where n_1 and n_2 are the electron density in the two layers and $1/4\kappa$ is the capacitance per unit area between the two layers. (Here we have assumed that the intralayer interaction is stronger than the interlayer interaction so that κ is positive.)

The above discussion suggests that, in the absence of interlayer hopping, a double-layer QH state $\Psi_{(mmm)}$ at filling fraction $\nu = 1/m$ support a gapless mode of collective excitations. The gapless mode is associated with strong fluctuations of $n_1 - n_2$. Notice that in the absence of interlayer hopping there are two $U(1)$ symmetries, one associated with the conservation of the total electric charge $N_1 + N_2$, the other with the conservation of $N_1 - N_2$. Therefore the gapless mode is the NG mode arising from the spontaneous broken $U(1)$ symmetry associated with $N_1 - N_2$, characterized by off-diagonal long range order in the hopping operator $c_1^\dagger c_2$. (Here $c_{1,2}$ is the electron operators in the two layers.) In the presence of interlayer hopping, the quantity $N_1 - N_2$ is no longer conserved and consequently we expect a finite energy gap.

An effective theory of the Hall fluid was developed based on the Lagrangian[8]

$$\mathcal{L} = \frac{1}{4\pi}(\sum_{IJ} K_{IJ}\alpha_I^\mu \partial^\nu \alpha_J^\lambda \epsilon_{\mu\nu\lambda} + \sum_I 2A^\mu \partial^\nu \alpha_I^\lambda \epsilon_{\mu\nu\lambda}) + \text{Maxwell terms} \tag{5}$$

involving the gauge potential α_I, $I = 1, ..., l$. Long distance properties of the Hall fluid are determined by the symmetric integer matrix K. In particular, the filling fraction is given by $\nu = \sum_{IJ}(K^{-1})_{IJ}$. This formalism is particularly suitable for describing multi-layer systems, with the electromagnetic current in the I^{th} layer given by $J_\mu^I = \frac{1}{2\pi}\partial^\nu \alpha_I^\lambda \epsilon_{\mu\nu\lambda}$, with the time component the electron density in each layers: $J_0^I = n_I$. Here we focus on two-layer systems with $K = \begin{pmatrix} l & n \\ n & m \end{pmatrix}$, corresponding to the wave function in (1), and with the filling fraction $\nu = \frac{l+m-2n}{lm-n^2}$.

When K has a zero eigenvalue, the corresponding linear combination of the gauge fields become massless, with dynamics governed by Maxwell terms in (5). For $K = m\begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix}$ (giving $\nu = 1/m$ as we can see by taking a suitable limit), the combination $\alpha_+ = \alpha_1 + \alpha_2$ has finite gap and couples to the electromagnetic potential A_μ thus describing a Hall fluid, while the combination $\alpha_- = \alpha_1 - \alpha_2$ is gapless with a linear dispersion, describing a superfluid associated with fluctuations in $n_1 - n_2$. Note that α_- decouples from the electromagnetic potential and represents neutral excitations. Notice that $n_1 - n_2 = \frac{1}{2\pi} f_{-,12}$. From (4) we see that the the Maxwell term for α_- is given by

$$\frac{\kappa}{8\pi^2}\left[-(f_{-,12})^2 + \frac{1}{v^2}(f_{-,0i})^2\right] \tag{6}$$

where v is the velocity of the linear mode and $f_{-,\mu\nu}$ is the field strength of the gauge field $\alpha_{-,\mu}$. The above is valid even when the two wells are not symmetric.

Within the mean field effective theory, the appearance of the gapless mode may be attributed to the coherent fluctuation of flux and density.[9,10] Notice that the electrons in the second layer behave like flux tubes of $-m\Phi_0$ flux to the electrons in the first layer (where Φ_0 is the flux quantum). The electrons in the first layer see an effective magnetic field $B^* = B - mn_2\Phi_0 = (1 - m\nu_2)B$, where ν_1 and ν_2 (with $\nu_1 + \nu_2 = 1/m$) are the electron filling fractions in the two layers. Therefore the electrons in the first layer have an effective filling fraction $\nu_1^* = n_1\Phi_0/B^* = \frac{\nu_1}{1-m\nu_2} = 1$. This is why the electron wave function within the first layer is the $1/m$ Laughlin wave function, $\prod(z_i - z_j)^m$. Similarly $\nu_2^* = 1/m$. As we move electron from the second layer to the first layer, n_1 increases, but at the same time B^* also increases since n_2 decreases. The effective filling fraction ν_1^* remain unchanged. Using similar arguments as in anyon superconductivity,[9] we see that such coherent fluctuation of flux and density gives rise to a gapless mode. Note that the correlation between electrons can only restrict the fluctuations of the filling fraction. In the ordinary QH system the magnetic field is fixed, and thus a density fluctuation will change the filling fraction and hence has a finite energy gap. In our case the effective magnetic field and density can fluctuate together without changing the (effective) filling fraction and lead to gapless excitations.

Because the linear gapless mode in the absence of interlayer hopping indicates the spontaneous breaking of the $U(1)$ symmetry associated with the conservation of $N_1 - N_2$, we expect a phase transition (of Kosterlitz-Thouless type) at finite temperature. A vortex in the superfluid of $N_1 - N_2$ is described by a particle that carries an unit charge of α_- gauge field.[9,11,12] A vortex-anti-vortex pair has an energy $\frac{2\pi v^2}{\kappa} \ln |r_1 - r_2|$. From this we can easily determine the critical temperature $T_c \approx \frac{\pi v^2}{2\kappa}$. The above result is just an estimate because at finite temperatures v and κ may have different values than the zero-temperature ones. When the interlayer separation is of order magnetic length, $e^2/\epsilon l_B$ is the only energy scale in the problem which determines both T_c and the charged-quasiparticle gap. Thus we expect T_c is of order of the charged-quasiparticle gap. In the low temperature phase, the operator $c_1^\dagger c_2$ has a long range correlation which decays algebricly. In high temperature phase, the correlation of $c_1^\dagger c_2$ is short ranged.

At finite interlayer hopping, $N_1 - N_2$ is no longer conserved and the phase transition is expected to be smeared into a cross over behavior.

Thus far the formalism does not incorporate interlayer hopping. When an electron hops from one layer to another, the currents J_1^μ and J_2^μ are no longer separately conserved. Indeed,

$$\int dt d^2x \partial_\mu (J_1^\mu - J_2^\mu) = \frac{1}{2\pi} \int dt d^2x \partial_\mu (\epsilon^{\mu\nu\lambda} \partial_\nu \alpha_{-,\lambda}) = \pm 2 \tag{7}$$

indicating the presence in Euclidean spacetime of a Dirac monopole (or antimonopole), that is, an instanton. Amusingly, in this formalism, interlayer electron hopping corresponds to the instanton described by a monopole in the gauge potential α_-. This makes sense because the electrons in layer I correspond to the flux quanta associated with α_I. The monopole turns flux_2 into flux_1. We have shown previously that monopoles convert anyon superfluid into normal fluid.[9]

Now the dynamics of the gauge potential α_- has to be considered within a monopole-antimonopole plasma. Polyakov[13] showed long ago that non-perturbative effects in the plasma generate a energy gap for α_-. The effective theory for the neutral low lying excitations can be obtained following the calculations in Ref. 13

$$\begin{aligned} \mathcal{L}_{eff} = & -\frac{1}{2}\frac{1}{\zeta}(\partial_\mu J_{-,\mu})^2 + \frac{1}{2}\kappa (J_{-,\mu})^2 + \frac{1}{2}\gamma \epsilon^{\mu\nu\lambda} J_{-,\mu} \partial_\nu J_{-,\lambda} \\ & + \beta \epsilon^{\mu\nu\lambda} J_{-,\mu} F_{\nu\lambda} + \frac{\sigma_{xy}}{4\pi} \epsilon^{\mu\nu\lambda} A_\mu \partial_\nu A_\lambda \end{aligned} \tag{8}$$

in units with $v = 1$. Here $J_{-,\mu} = J^1_\mu - J^2_\mu$ (*e.g.,* $J_{-,0} = n_1 - n_2$) is the difference of the electron densities and currents in the two layers. The first term is due to the monopole plasma (*i.e.,* the interlayer hopping of electrons). ζ is the probability of finding a monopole in a unit spacetime volume (ζ is proportional to the interlayer tunneling rate) and κ is the coefficient in (6). We estimate ζ to be of order Δ_{SAS}/l_B^2, where Δ_{SAS} is the energy gap between the symmetric and the antisymmetric states in the double wells (Δ_{SAS} is proportional to the interlayer hopping amplitude), and l_B the magnetic length. The last three terms violate time reversal symmetry and parity and are induced by integrating out α_+. The parameter γ, β are independent of the interlayer hopping. From (8) we can calculate the correlations between $J_{-,\mu}$'s:

$$\langle J_{-,\mu} J_{-,\nu} \rangle = \frac{\frac{\kappa}{\gamma^2}}{\frac{\kappa^2}{\gamma^2} + \vec{k}^2 - \omega^2} \Big(g_{\mu\nu} - \frac{k_\mu k_\nu}{k^2} \Big) + \frac{\frac{\Delta^2}{\kappa}}{\Delta^2 + \vec{k}^2 - \omega^2} \frac{k_\mu k_\nu}{k^2} + i \frac{\gamma^{-1} \epsilon_{\mu\nu\lambda} k^\lambda}{\frac{\kappa^2}{\gamma^2} + \vec{k}^2 - \omega^2} \tag{9}$$

where $\Delta = \sqrt{\zeta\kappa} \sim \sqrt{\frac{\Delta_{SAS}\kappa}{l_B^2}}$. From (9) we see that there is a low lying mode with gap Δ. (Note $\Delta \to 0$ as the interlayer hopping approaches zero.) This low lying excitation is the longitudinal mode $k_\mu J_{-,\mu}$. The transverse modes satisfying $k_\mu J_{-,\mu} = 0$ have a large gap κ/γ. At this energy scale the effective theory (8) may not be reliable. (Note that there is no pole at $k^2 = 0$.) We would like to emphasize that we only have a single low energy mode. The transverse modes have no dynamics at low energies (*e.g.,* they do not contribute to the low temperature specific heat). Using the effective theory (8) and the correlation function (9) we can study various properties of the low energy mode. Since the low energy excitations described by $J_{-,\mu}$ only couple to the field strength $F_{\mu\nu}$ instead of the gauge potential A_μ, they have no net electric charge, but they have some magnetic and/or electric dipole moments if the symmetry between the two layers is broken. (Note $\beta = 0$ if there is a symmetry between the two layers. In this case the effective Lagrangian (8) must be invariant under $J_- \to -J_-$ and J_- cannot have a linear coupling to A_μ.) Putting back the velocity v, we have $\Delta \sim \sqrt{\frac{\Delta_{SAS}\kappa}{l_B^2}}$, independent of v, giving the dispersion relation of $E_k = \sqrt{v^2 k^2 + \Delta^2}$ for neutral low lying mode.

Using the formalism described in Ref. 9 we can readily determine the charge and statistics of the quasiparticles and verify that in the presence of the gap the fluid described here is in the same universality class as the $\nu = 1/m$ Laughlin state.

An electric field $\mathcal{E}$ perpendicular to the plane $\mathcal{E}$ will generate a dipole moment due to charge imbalance in the two layers. The induced dipole density is given by $D = e(n_1 - n_2)d \equiv \chi\mathcal{E}$, where d is the separation between the two layers. The coupling between $J_{-,\mu}$ and $\mathcal{E}$ is described by $(ed)\mathcal{E}J_{-,0}$. From (9) we see that, at frequency ω and wave number k, the susceptibility χ is given by

$$\chi(\omega, k) = \Big(\frac{\Delta^2 + v^2 k^2}{\Delta^2 + v^2 k^2 - \omega^2} \Big) \frac{e^2 d^2}{\kappa} \tag{10}$$

for small (ω, k) and Δ. Thus the the neutral excitations can be observed through the resonance at $\omega = \sqrt{\Delta^2 + v^2 k^2}$, with an (integrated) strength proportional to $(e^2 d^2/\kappa)\sqrt{\Delta^2 + v^2 k^2}$. The effective Lagrangian also contains a term $\sim \mathcal{E}B$ which can be detected, at least in principle, in the propagation of polarized light. Taking into account the angular character of the order parameter, we find[14] that a voltage V applied across the two layers generates a tunnelling current proportional to $\sin \frac{eVt}{\hbar}$.

If we can attach leads to the individual layers, then we can generate electric fields with opposite directions in the two layers (*i.e.,* $\vec{E}_- = \vec{E}_1 - \vec{E}_2 \neq 0$). The pseudoconductivity defined by $\vec{J}_- = \sigma_- \vec{E}_-$ is given by the $0i$ component of the J_-−current correlation (divided by $i2\vec{k}$)

$$\sigma_-(\omega) = \frac{\hbar v^2}{2\kappa} \frac{i\omega}{\Delta^2 - \omega^2} \frac{e^2}{\hbar} \tag{11}$$

Note when $\Delta = 0$ the above reduces to the conductivity of a superconductor.

When the intralayer and interlayer interactions are equal and in the absence of hopping, the system has an additional symmetry.[15,16] The presence of the gapless mode can also be derived from this symmetry. To describe the higher symmetry, let us view the electrons in the two layers as a pseudospin doublet (*i.e.,* the electrons on layer 1 carry pseudospin $S_z = 1/2$ and layer 2 $S_z = -1/2$). The interaction is invariant under pseudospin rotation. Thus in addition to the $U(1)$ symmetry associated with the conservation of the electric charge, the system also has pseudospin rotational symmetry, part of which is the $U(1)$ symmetry associated with $N_1 - N_2$. For the Coulomb interaction, the ground state at $\nu = 1$ carry maximum pseudospin $S = N_e/2$ and the electrons from a ferromagnetic state of the pseudospins.[15,17] Therefore there exist gapless excitations corresponding to the spin wave in the ferromagnetic state, with quadratic dispersion relation $\omega \propto k^2$. We expect that the spin waves are the only low energy excitations above the ground state $\Psi_{(111)}$. All the low energy excitations (*i.e.,* the spin waves) are descried by the effective Hamiltonian

$$H = -J \sum (S_x^i S_x^j + S_y^i S_y^j + \eta S_z^i S_z^j) \tag{12}$$

where $\vec{S}^i$ is the pseudospin operator of the electrons, and we imagine latticzing the plane with spacing of the order magnetic length. When the interlayer interaction is equal to the intralayer interaction $\eta = 1$ due to the $SU(2)$ symmetry. When the intralayer interaction and the interlayer interaction are not equal to each other, the pseudospin rotation symmetry will be broken down to the $U(1)$ symmetry associated with $N_1 - N_2$. If the intralayer interaction is stronger than the interlayer one (which is the case in real samples), η will be less than 1 (note $S_z \sim n_1 - n_2$ and $1 - \eta \sim$ intralayer-interaction − interlayer-interaction). In this case, we have an XY model with linear dispersion at small k, consistent with the effective theory analysis. Interlayer hopping corresponds to adding a magnetic field in the x-direction, that is adding a term $\Delta_{SAS} \sum S_x^i$ to the Hamiltonian since the hopping is given by the pseudospin operator $2S_x$. Thus it is easy to understand that hopping will open an energy gap proportional to $\sqrt{\Delta_{SAS}}$. When the interlayer interaction is stronger then the intralayer one, we have $\eta > 1$ and the low energy dynamics will be described by the Ising model. The electrons in the ground state occupy only one of the two layers.

Many numerical calculations for the double layer system at filling fraction $\nu = 1$ have been performed.[18,15,19] In Ref. 18,15 the electron interaction is chosen to be

$$V^{\sigma\sigma'} = \frac{e^2}{\epsilon(r^2 + d^2 \delta_{\sigma,-\sigma'})^{1/2}} \tag{13}$$

where $\sigma, \sigma' = \pm 1$ indicate the two layers and d can be viewed as the interlayer separation. When $d/l_B < 0.5$, $\Psi_{(111)}$ was shown to have a large overlap with the exact ground state wave function of finite electron systems at $\nu = 1$. This result, combined with the results in this paper, suggests that the ground state support a gapless collective mode with a linear dispersion at least when $d/l_B < 0.5$. Our results appear to contradict with the numerical results in Ref. 18, where a finite energy gap is found at $d/l_B = 2$ and $\nu = 1$. However, the calculation in Ref. 18 is performed with fixed number of electrons in each layer. In this case it is difficult to see the gapless excitations. Therefore it might be possible that the system studied in Ref. 18 is too small to see the gapless mode.

Because the low energy mode is neutral, it will not affect electric transport properties. The QH plateau will not be destroyed by the presence of neutral low lying excitations. (Our results do not explain the collapse of the odd-integer QH states observed in Ref. 1.) In Ref. 3, a strong $\nu = 1$ QH state was observed in samples with small interlayer hoppings. Therefore, this state may contain a low lying neutral excitation with the dispersion given in (9). Because the interlayer separation is of order of the magnetic length, the velocity is of order $e^2/\epsilon\hbar = \frac{c}{137\epsilon}$. It would be interesting to detect superfluidity behavior experimentally in this system. The discussion in this paper may be extended readily to multi-layer systems with finite filling fraction and described[8] by a matrix K with zero determinant.

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