Origin of Light

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The existence of light (a *massless* $U(1)$ gauge boson) is one of unresolved mysteries in nature. In this paper, we would like to propose that light is originated from certain quantum orders in our vacuum. We will construct quantum spin models on lattice to demonstrate that some quantum orders can give rise to light without breaking any symmetries and without any fine tuning. Through our models, we show that the existence of light can simply be a phenomenon of quantum coherence in a system with many degrees of freedom. Massless gauge fluctuations appears commonly and naturally in strongly correlated *quantum* systems which originally contain no gauge fields.

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In an attempt to explain the meaning of "empty space" to a young child, I said "space is something not made of atoms." He replied "Then you were wrong to tell me last time that only light is not made of atoms." Indeed, light and gravity are two singular forms of "matter" which are very different from other forms of matter such as atoms, electrons, *etc* . (Here I assume space = gravity.) The existences of light and gravity – two massless gauge bosons – are two big mysteries in nature.

Massless particles are very rare in nature. In fact photon and graviton are the only two massless particles known to exist. In condensed matter systems, one encounters more kinds of gapless excitations. However, with a few exceptions, all the gapless excitations exist because the ground state of the system has a special property called spontaneous breaking of a continuous symmetry.[1, 2] For example, gapless phonons exist in a solid because a solid break the continuous translation symmetries. There are precisely three kinds of gapless phonons since the solid breaks three translation symmetries in x, y and z directions. Thus we can say that the origin of gapless phonons is the translation symmetry breaking in solids.

With the above understanding of the origin of gapless phonon in solids, we would like to ask what is the origin of light? Here we will adopt a point of view that all particles, such as photons, electrons, *etc* , are excitations above a ground state – the vacuum. The properties of those particles reflect the properties of the vacuum. With this point of view, the question on the origin of light become a question on the properties of vacuum that allow and protect the existence of light.

If light behaved like phonons in solids, then we could conclude that our vacuum break a continuous symmetry and light would be originated from symmetry breaking. However, in reality, light does not behave like the phonons. In fact there are no phonon-like particles (or more precisely, massless Nambu-Goldstone bosons) in nature. From the lack of massless Nambu-Goldstone bosons, we can conclude that there is no continuous symmetry breaking in our vacuum. If the vacuum does not break any continuous symmetry, then what makes light to exist?

In a recent work,[3, 4] a concept – quantum order – was introduced to describe a new kind of orders that generally appear in quantum states at zero temperature. Quantum orders that characterize universality classes of quantum states (described by *complex* ground state wave functions) is much richer then classical orders that characterize universality classes of finite temperature classical states (described by *positive* probability distribution functions). In contrast to classical orders, quantum orders cannot be described by broken symmetries and the associated order parameters. A new mathematical object – projective symmetry group (PSG) – was introduced to characterize quantum orders. In a sense, we can view a quantum order as a dancing pattern in which particles waltz around each other in a ground state. The PSG is a mathematical description of the dancing pattern. In contract, the classical order in a crystal just describes a static positional pattern, which can be characterized by symmetries.

In Ref. [3], various quantum orders are studied. It was found that different quantum orders (characterized by different PSG's) can have distinct low energy properties. In particular, certain quantum orders allow and protect gapless excitations even without breaking any continuous symmetry. This leads us to propose that it is the quantum order in our vacuum that allow and protect the existence of light. In another word, light is originated from quantum order.

To support our idea, in the following, we are going to study a concrete $SU(N_f)$ spin model[5, 6] in 3D, and show that its ground state contains a gapless collective fluctuation given by Eq. (12) which behaves in every way like a $U(1)$ gauge fluctuation. More importantly, we will identify the quantum order (or the PSG) in the ground state and argue that the gapless property of the $U(1)$ gauge fluctuations is a robust property protected by the quantum orders. A small change of the Hamiltonian cannot destroy the gapless $U(1)$ gauge fluctuations. We would like to mention that a connection between QCD and a

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lattice spin model was pointed out in Ref. [7], using concept of quantum critical point. In our example, we will see that the massless property of light is not due to criticality. It is a generic property of a quantum phase.

We start with a $SU(N_f)$-spin model[5, 6] on a 3D cubic lattice. The states on each site form a representation of rank $N_f/2$ antisymmetric tensor of $SU(N_f)$. We note that those states can be viewed as states of $N_f/2$ fermions with fermions $\psi_{a\boldsymbol{i}}$, $a = 1, ..., N_f$ in the fundamental representation of $SU(N_f)$. Thus we can write down the Hamiltonian of our model in terms of the fermion operators:

$$H = J_P \sum_{\langle \boldsymbol{i}_1 \boldsymbol{i}_2 \boldsymbol{i}_3 \boldsymbol{i}_4 \rangle} \left(S^{ab}_{\boldsymbol{i}_1} S^{bc}_{\boldsymbol{i}_2} S^{cd}_{\boldsymbol{i}_3} S^{da}_{\boldsymbol{i}_4} + h.c. \right) \quad (1)$$

where the sum is over all plaquettes $\langle \boldsymbol{i}_1 \boldsymbol{i}_2 \boldsymbol{i}_3 \boldsymbol{i}_4 \rangle$,

$$S^{ab}_{\boldsymbol{i}} = \psi^\dagger_{a\boldsymbol{i}} \psi_{b\boldsymbol{i}} - N_f^{-1} \delta_{ab} \psi^\dagger_{c\boldsymbol{i}} \psi_{c\boldsymbol{i}}. \quad (2)$$

The Hamiltonian has three translation symmetries and six parity symmetries $P_x : x \to -x$, $P_y : y \to -y$, $P_z : z \to -z$, $P_{xy} : x \leftrightarrow y$, $P_{yz} : y \leftrightarrow z$, $P_{zx} : z \leftrightarrow x$. The Hamiltonian also has a charge conjugation symmetry $C : \psi_{a\boldsymbol{i}} \to \psi^\dagger_{a\boldsymbol{i}}$.

To find the ground state of the above systems, we will use the projective construction (which is a generalization of slave-boson approach[5, 8, 9]) to construct the ground state. We start with a mean-field parton Hamiltonian

$$H_{mean} = -\sum_{\langle \boldsymbol{ij} \rangle} \left(\psi^\dagger_{a,\boldsymbol{i}} \chi_{\boldsymbol{ij}} \psi_{a,\boldsymbol{j}} + h.c. \right) \quad (3)$$

where $\chi^\dagger_{\boldsymbol{ij}} = \chi_{\boldsymbol{ji}}$. The mean-field Hamiltonian allows us to construct a trial wave function for the ground state of the $SU(N_f)$-spin system Eq. (1):

$$|\Psi^{(\chi_{\boldsymbol{ij}})}_{trial}\rangle = \mathcal{P} |\Phi^{(\chi_{\boldsymbol{ij}})}_{mean}\rangle \quad (4)$$

where $|\Phi^{(\chi_{\boldsymbol{ij}})}_{mean}\rangle$ is the ground state of the mean-field Hamiltonian H_{mean} and $\mathcal{P}$ is the projection to states with $N_f/2$ fermion per site. Clearly the mean-field ground state is a functional of $\chi_{\boldsymbol{ij}}$. The proper values of $\chi_{\boldsymbol{ij}}$ are obtained by minimizing the trial energy $E = \langle \Psi^{(\chi_{\boldsymbol{ij}})}_{trial} | H | \Psi^{(\chi_{\boldsymbol{ij}})}_{trial} \rangle$.

The relation between the physical operator S^{ab} and the parton operator ψ_a essentially defines the projective construction.[10] For example the fact that the operator $S^{ab}_{\boldsymbol{i}}$ is invariant under local $U(1)$ transformations

$$\psi_{a\boldsymbol{i}} \to e^{i\theta_{\boldsymbol{i}}} \psi_{a\boldsymbol{i}}, \qquad S^{ab}_{\boldsymbol{i}} \to S^{ab}_{\boldsymbol{i}} \quad (5)$$

determines the high energy $U(1)$ gauge structure in the parton mean-field theory:

$$\psi_{a\boldsymbol{i}} \to e^{i\theta_{\boldsymbol{i}}} \psi_{a\boldsymbol{i}}, \qquad \chi_{\boldsymbol{ij}} \to e^{i\theta_{\boldsymbol{i}}} \chi_{\boldsymbol{ij}} e^{-i\theta_{\boldsymbol{i}}} \quad (6)$$

The $U(1)$ gauge structure has a very real meaning: two gauge equivalent ansatz give rise to the same physical state after projection

$$|\Psi^{(\chi_{\boldsymbol{ij}})}_{trial}\rangle = |\Psi^{(e^{i\theta_{\boldsymbol{i}}} \chi_{\boldsymbol{ij}} e^{-i\theta_{\boldsymbol{j}}})}_{trial}\rangle \quad (7)$$

Usually it is hard to calculate the trial energy $E = \langle \Psi_{trial} | H | \Psi_{trial} \rangle$. In the following, we will calculate $\chi_{\boldsymbol{ij}}$ by minimizing the mean-field energy $E_{mean} = \langle \Phi^{(\chi_{\boldsymbol{ij}})}_{mean} | H | \Phi^{(\chi_{\boldsymbol{ij}})}_{mean} \rangle$ which approaches to the exact ground state energy in the large N_f limit.[5, 6] We assume $|\Phi^{(\chi_{\boldsymbol{ij}})}_{mean}\rangle$ to respect the $SU(N_f)$ symmetry, which leads to $\langle \Phi^{(\chi_{\boldsymbol{ij}})}_{mean} | \psi_{a\boldsymbol{i}} \psi^\dagger_{b\boldsymbol{j}} | \Phi^{(\chi_{\boldsymbol{ij}})}_{mean} \rangle = \delta_{ab} \tilde{\chi}_{\boldsymbol{ij}}$. We find

$$\frac{E_{mean}}{J_P N_f^4} = \sum_{\langle \boldsymbol{i}_1 \boldsymbol{i}_2 \boldsymbol{i}_3 \boldsymbol{i}_4 \rangle} (\tilde{\chi}_{\boldsymbol{i}_1\boldsymbol{i}_2} \tilde{\chi}_{\boldsymbol{i}_2\boldsymbol{i}_3} \tilde{\chi}_{\boldsymbol{i}_3\boldsymbol{i}_4} \tilde{\chi}_{\boldsymbol{i}_4\boldsymbol{i}_1} + h.c.) + O(N_f^{-1})$$

Since a π-flux in a plaquette make $\tilde{\chi}_{\boldsymbol{i}_1\boldsymbol{i}_2} \tilde{\chi}_{\boldsymbol{i}_2\boldsymbol{i}_3} \tilde{\chi}_{\boldsymbol{i}_3\boldsymbol{i}_4} \tilde{\chi}_{\boldsymbol{i}_4\boldsymbol{i}_1}$ to be a negative number, we expect the ansatz that minimize E_{mean} to have π flux on every plaquette. Such an ansatz can be constructed and takes a form[11]

$$\begin{aligned} \bar{\chi}_{\boldsymbol{i},\boldsymbol{i}+\hat{\boldsymbol{x}}} &= -i\chi, \qquad \bar{\chi}_{\boldsymbol{i},\boldsymbol{i}+\hat{\boldsymbol{y}}} = -i(-)^{i_x}\chi, \\ \bar{\chi}_{\boldsymbol{i},\boldsymbol{i}+\hat{\boldsymbol{z}}} &= -i(-)^{i_x+i_y}\chi. \end{aligned} \quad (8)$$

Such an ansatz, after projection, gives rise to a correlated ground state for our $SU(N_f)$-spin system.

In the momentum space, the mean-field Hamiltonian has a form

$$H_{\rm mean} = -{\sum_{\boldsymbol{k}}}' \Psi^\dagger_{a,\boldsymbol{k}} \Gamma(\boldsymbol{k}) \Psi_{a,\boldsymbol{k}} \quad (9)$$

where

$$\begin{aligned} \Psi^T_{a,\boldsymbol{k}} =& (\psi_{a,\boldsymbol{k}}, \psi_{a,\boldsymbol{k}+\boldsymbol{Q}_x}, \psi_{a,\boldsymbol{k}+\boldsymbol{Q}_y}, \psi_{a,\boldsymbol{k}+\boldsymbol{Q}_x+\boldsymbol{Q}_y}), \\ \boldsymbol{Q}_x =& (\pi, 0, 0), \quad \boldsymbol{Q}_y = (0, \pi, 0), \\ \Gamma(\boldsymbol{k}) =& 2\chi(\sin(k_x)\Gamma_1 + \sin(k_y)\Gamma_2 + \sin(k_z)\Gamma_3) \end{aligned}$$

and $\Gamma_1 = \tau^3 \otimes \tau^0$, $\Gamma_2 = \tau^1 \otimes \tau^3$, and $\Gamma_3 = \tau^1 \otimes \tau^1$. The momentum summation is over a range $k_x \in (-\pi/2, \pi/2)$, $k_y \in (-\pi/2, \pi/2)$, and $k_z \in (-\pi, \pi)$. Since $\{\Gamma_i, \Gamma_j\} = 2\delta_{ij}$, $i, j = 1, 2, 3$, we find partons have a dispersion

$$E(\boldsymbol{k}) = \pm 2\chi \sqrt{\sin^2(k_x) + \sin^2(k_y) + \sin^2(k_z)} \quad (10)$$

The mean-field ground state $|\Phi_{mean}\rangle$ is obtained by filling the negative energy branch. We see that the dispersion has two nodes at $\boldsymbol{k} = 0$ and $\boldsymbol{k} = (0, 0, \pi)$. Thus there are $2N_f$ massless four-component Dirac fermions in the continuum limit. The low energy theory has Lorentz symmetry. Including the collective phase fluctuations of the ansatz, the low energy effective theory has a form

$$L = \sum_{\boldsymbol{i}} \psi^\dagger_{a,\boldsymbol{i}} i(\partial_t + ia_0) \psi_{a,\boldsymbol{j}} + \sum_{\boldsymbol{ij}} \psi^\dagger_{a,\boldsymbol{i}} \bar{\chi}_{\boldsymbol{ij}} e^{ia_{ij}} \psi_{a,\boldsymbol{j}}$$

In the continuum limit, it becomes $\mathcal{L} = \bar{\psi}_{\alpha a} D_\mu \gamma^\mu \psi_{\alpha a}$ with $D_\mu = \partial_\mu + ia_\mu$, $\alpha = 1, 2$, and γ_μ are 4×4 Dirac matrices.[11] Integrating out the high energy fermions will generate dynamics for the a_μ field (see Eq. (17)). We see that our correlated ground state, $\mathcal{P}|\Phi^{(\bar{\chi}_{\boldsymbol{ij}})}_{mean}\rangle$, support a massless $U(1)$ gauge fluctuations and $2N_f$ massless Dirac fermions.

In the following we would like to argue that the appearance of the massless $U(1)$ gauge fluctuations and the massless Dirac fermions is not a special property of the particular state constructed above. It is a universal property of a quantum phase (characterized by a particular quantum order). We will first find the PSG of the constructed state. Then we will argue that the PSG is a universal property of a quantum phase by showing that radiative corrections cannot change the PSG. Last we will show that any state described by the same PSG (*ie* any state in the same quantum phase) will have the same massless $U(1)$ gauge fluctuations and the same massless Dirac fermions. We would like to remark that the stability of the massless $U(1)$ gauge fluctuations in 3+1D is not new. But the stability of massless Dirac fermions is new and *the PSG approach puts the stability of the massless* $U(1)$ *gauge fluctuations and the stability of massless Dirac fermions on the same footing.*

The PSG[3, 4] that characterizes the quantum order in the above correlated state is given by

$$\begin{aligned} G_x(\boldsymbol{i}) &=(-)^{i_y+i_z}e^{i\theta_x} & G_y(\boldsymbol{i}) &=(-)^{i_z}e^{i\theta_y} \\ G_z(\boldsymbol{i}) &=e^{i\theta_z} & G_{px}(\boldsymbol{i}) &=(-)^{i_x}e^{i\theta_{px}} \\ G_{py}(\boldsymbol{i}) &=(-)^{i_y}e^{i\theta_{py}} & G_{pz}(\boldsymbol{i}) &=(-)^{i_z}e^{i\theta_{pz}} \\ G_{pxy}(\boldsymbol{i}) &=(-)^{i_x i_y}e^{i\theta_{pxy}} & G_{pyz}(\boldsymbol{i}) &=(-)^{i_y i_z}e^{i\theta_{pyz}} \\ G_C(\boldsymbol{i}) &=(-)^{\boldsymbol{i}}e^{i\theta_t} & G_{pzx}(\boldsymbol{i}) &=(-)^{(i_x+i_y)(i_y+i_z)}e^{i\theta_{pzx}} \end{aligned} \tag{11}$$

The invariant gauge group (IGG) of the ansatz is $\mathcal{G}=\{e^{i\theta}\}=U(1)$, which is a (normal) subgroup of the PSG. Here $G_{x,y,z}$ are the gauge transformations associated with the three translations, $G_{px,py,pz}$ are associated with the three parities P_x, P_y, P_z, and $G_{pxy,pyz,pzx}$ are associated with the other three parities P_{xy}, P_{yz}, P_{zx}, and G_C is associated with charge conjugation transformation $C:\chi_{\boldsymbol{ij}}\to-\chi_{\boldsymbol{ij}}$. The ansatz is invariant, say, under the parity transformation P_x followed by the gauge transformation G_{px}.

To show that the PSG is a universal property of a quantum phase,[3] we start with the mean-field state characterized by $\chi_{\boldsymbol{ij}}=N_f^{-1}\langle\psi_{a\boldsymbol{i}}\psi^\dagger_{a\boldsymbol{j}}\rangle$. If we include perturbative fluctuations around the mean-field state, we expect $\chi_{\boldsymbol{ij}}$ to receive radiative corrections $\delta\chi_{\boldsymbol{ij}}$. However, the perturbative fluctuations can only change $\chi_{\boldsymbol{ij}}$ in such a way that $\chi_{\boldsymbol{ij}}$ and $\chi_{\boldsymbol{ij}}+\delta\chi_{\boldsymbol{ij}}$ have the same projective symmetry group. This is because if $\chi_{\boldsymbol{ij}}$ and the Hamiltonian have a symmetry, then $\delta\chi_{\boldsymbol{ij}}$ generated by perturbative fluctuations will have the same symmetry. The transformation generated by an element in PSG just behave like a symmetry transformation in the perturbative calculation. The mean-field ground state and the mean-field Hamiltonian are invariant under the transformations in the PSG. Therefore, $\delta\chi_{\boldsymbol{ij}}$ generated by perturbative fluctuations will also be invariant under the transformations in the PSG. Thus the perturbative fluctuations cannot change the PSG of an ansatz. Also if we perturb the $SU(N_f)$-spin Hamiltonian Eq. (1) without breaking any symmetries, the induced $\delta\chi_{\boldsymbol{ij}}$ is still invariant under the transformations in the PSG. Thus the PSG is robust against small perturbations of the Hamiltonian and it is a universal property of a quantum phase. The PSG can change only when the fluctuations have an infrared divergence which will drive a phase transition. From Eq. (18), we see that the coupling between the $U(1)$ gauge field and the massless Dirac fermions is irrelevant at low energies. Thus there is no infrared divergence in our model and the interaction between fermions and gauge field cannot make the gauge field and fermions massive (see below).

To understand how quantum orders and PSG's protect the gapless excitations without breaking any symmetries, we would like to first find out the possible fluctuations at low energies. The first kind of low energy excitations are described by the particle-hole excitations of the fermions across the Fermi points. The $SU(N_f)$-spin wave-functions for such kind of excitations are given by $|\Psi^{(\bar\chi_{ij})}_{exc}\rangle=\mathcal{P}\psi^\dagger_{\boldsymbol{k}_1}\psi_{\boldsymbol{k}_2}|\Phi^{(\bar\chi_{ij})}_{mean}\rangle$. The second kind of low energy excitations are the collective excitations described by the phase fluctuations of the ansatz: $\chi_{\boldsymbol{ij}}=\bar\chi_{\boldsymbol{ij}}e^{ia_{\boldsymbol{ij}}}$. The $SU(N_f)$-spin wave-functions for such collective excitations are given by

$$|\Psi^{(\bar\chi_{\boldsymbol{ij}}e^{ia_{\boldsymbol{ij}}})}_{exc}\rangle=\mathcal{P}|\Phi^{(\bar\chi_{\boldsymbol{ij}}e^{ia_{\boldsymbol{ij}}})}_{mean}\rangle. \tag{12}$$

To see that the massless fermion excitations are protected by the quantum order, we need to consider the most generic ansatz $\chi_{\boldsymbol{ij}}$ that have the same PSG Eq. (11) and check if the fermions are still massless for those generic ansatz. The most general translation symmetric ansatz has a form

$$\chi_{\boldsymbol{i},\boldsymbol{i}+\boldsymbol{m}}=\chi_{\boldsymbol{m}}(-)^{i_y m_z}(-)^{i_x(m_y+m_z)} \tag{13}$$

To have the parity symmetry $\boldsymbol{i}\to-\boldsymbol{i}$, the ansatz should be invariant under transformation $\boldsymbol{i}\to-\boldsymbol{i}$ followed by a gauge transformation $(-)^{\boldsymbol{i}}$. This requires that $\chi_{\boldsymbol{m}}=(-)^{\boldsymbol{m}}\chi_{-\boldsymbol{m}}=(-)^{\boldsymbol{m}}\chi^\dagger_{\boldsymbol{m}}$. To have charge conjugation symmetry $\chi_{\boldsymbol{ij}}$ must change sign under gauge transformation $W_{\boldsymbol{i}}=(-)^{\boldsymbol{i}}$. This requires that $\chi_{\boldsymbol{m}}=0$, if $\boldsymbol{m}=even$. Thus the most general ansatz has a form

$$\begin{aligned} \chi_{\boldsymbol{i},\boldsymbol{i}+\boldsymbol{m}} &=\chi_{\boldsymbol{m}}(-)^{i_y m_x}(-)^{i_x(m_x+m_y)} \\ \chi_{\boldsymbol{m}} &=0, \quad \text{if } \boldsymbol{m}=even \\ \chi_{\boldsymbol{m}} &=-\chi^\dagger_{\boldsymbol{m}}=-\chi_{-\boldsymbol{m}} \end{aligned} \tag{14}$$

In the momentum space, χ vanishes at $\boldsymbol{k}=0$ and $(0,0,\pi)$. Thus the PSG protect the massless Dirac fermions.

To see that the massless collective fluctuations described by $a_{\boldsymbol{ij}}$ are protected by the quantum order, we need to show the collective fluctuations are massless for the most general ansatz that have the same PSG Eq. (11). For any ansatz that is invariant on the PSG, it is also invariant under the IGG $\mathcal{G}=\{e^{i\theta}\}=U(1)$ which is subgroup of the PSG. In this case $a_{\boldsymbol{ij}}$ and $\tilde a_{\boldsymbol{ij}}=a_{\boldsymbol{ij}}+\theta_{\boldsymbol{i}}-\theta_{\boldsymbol{j}}$ label the same quantum state (and are said to be gauge equivalent). (See Eq. (7).) We see that $a_{\boldsymbol{ij}}$ describes a $U(1)$ gauge fluctuation. Since the energy of the fluctuation $E(a_{\boldsymbol{ij}})$ satisfies $E(a_{\boldsymbol{ij}}) = E(\tilde{a}_{\boldsymbol{ij}})$, the mass term $(a_{\boldsymbol{ij}})^2$ is not allowed and there is no Anderson-Higgs mechanism to give $U(1)$ gauge field a mass. Thus the $U(1)$ gauge fluctuations are gapless for any ansatz that has the PSG in Eq. (11).

In the standard analysis of the stability of the massless excitations, one needs to include all the counter terms that have the right symmetries into the Lagrangian, since those terms can be generated by perturbative fluctuations. Then we examine if those allowed counter terms can destroy the massless excitations or not. In our problem, we need to consider all the possible corrections to the mean-field ansatz. However, the new feature here is that it is incorrect to use the symmetry group to determine the allowed corrections. We should use PSG to determine the allowed corrections in our analysis of the stability of the massless excitations.

Next we consider a model that contains both massless and massive fermions. The mean-field Hamiltonian is

$$H_{mean} = -\sum_{\langle \boldsymbol{ij} \rangle} \left(\psi^\dagger_{a,\boldsymbol{i}} \chi_{\boldsymbol{ij}} \psi_{a,\boldsymbol{j}} + h.c. \right) \tag{15}$$
$$- \sum_{\langle \boldsymbol{ij} \rangle} \left(\lambda^\dagger_{\alpha,\boldsymbol{i}} \chi_{\boldsymbol{ij}} \tau^3 \lambda_{\alpha,\boldsymbol{j}} + h.c. \right) - \sum_{\boldsymbol{i}} \lambda^\dagger_{\alpha,\boldsymbol{i}} m \tau^1 \lambda_{\alpha,\boldsymbol{i}}$$

where $a = 1, ..., N_f$, $\alpha = 1, ..., N'_f$, λ_α is a doublet: $\lambda^T_\alpha = (\lambda^{(1)}_\alpha, \lambda^{(2)}_\alpha)$, and $\chi_{\boldsymbol{ij}}$ is given in Eq. (8). The model has a $U(1)$ gauge structure defined by the gauge transformation $\psi_{a,\boldsymbol{i}} \to e^{i\theta_{\boldsymbol{i}}} \psi_{a,\boldsymbol{i}}$, $\lambda_{a,\boldsymbol{i}} \to e^{i\theta_{\boldsymbol{i}}} \lambda_{a,\boldsymbol{i}}$, and $\chi_{\boldsymbol{ij}} \to e^{i(\theta_{\boldsymbol{i}} - \theta_{\boldsymbol{j}})} \chi_{\boldsymbol{ij}}$. Clearly, the model has a $SU(N_f) \times SU(N'_f)$ global symmetry. The gauge invariant physical operators are given by $\psi^\dagger_{a,\boldsymbol{i}} \psi_{b,\boldsymbol{i}}$, $\lambda^\dagger_{a',\boldsymbol{i}} \lambda_{b',\boldsymbol{i}}$, and $\psi^\dagger_{a,\boldsymbol{i}} \lambda_{a',\boldsymbol{i}}$

In the momentum space, the above H_{mean} becomes $H_{\text{mean}} = -\sum_{\boldsymbol{k}}{}' \Psi^\dagger_{a,\boldsymbol{k}} \Gamma(\boldsymbol{k}) \Psi_{a,\boldsymbol{k}} + \Lambda^\dagger_{a,\boldsymbol{k}} \tilde{\Gamma}(\boldsymbol{k}) \Lambda_{a,\boldsymbol{k}}$, where $\Lambda^T_{a,\boldsymbol{k}} = (\lambda_{a,\boldsymbol{k}}, \lambda_{a,\boldsymbol{k}+\boldsymbol{Q}_x}, \lambda_{a,\boldsymbol{k}+\boldsymbol{Q}_y}, \lambda_{a,\boldsymbol{k}+\boldsymbol{Q}_x+\boldsymbol{Q}_y})$, $\boldsymbol{Q}_x = (\pi, 0, 0)$, $\boldsymbol{Q}_y = (0, \pi, 0)$, $\tilde{\Gamma}(\boldsymbol{k}) = 2\chi(\sin(k_x)\tilde{\Gamma}_1 + \sin(k_y)\tilde{\Gamma}_2 + \sin(k_z)\tilde{\Gamma}_3) + m\tilde{\Gamma}_m$, and $\tilde{\Gamma}_1 = \tau^3 \otimes \tau^0 \otimes \tau^3$, $\tilde{\Gamma}_2 = \tau^1 \otimes \tau^3 \otimes \tau^3$, $\tilde{\Gamma}_3 = \tau^1 \otimes \tau^1 \otimes \tau^3$, and $\tilde{\Gamma}_m = \tau^0 \otimes \tau^0 \otimes \tau^1$. We see that there are $2N_f$ massless Dirac fermions and $4N'_f$ massive Dirac fermions in the continuum limit. Those fermions carry crystal momenta near $\boldsymbol{k} = 0$ and $\boldsymbol{k} = (0, 0, \pi)$. The PSG that characterizes the above mean-field state is still given by Eq. (11), which acts on both ψ and λ. Since IGG $= U(1)$, the fluctuations around the mean-field state contain a $U(1)$ gauge field at low energies. After including the $U(1)$ gauge field and in the continuum limit, the low energy effective theory takes a form

$$\mathcal{L} = \sum_{I=1}^{2N_f} \bar{\psi}_I D_\mu \gamma^\mu \psi_I + \sum_{J=1}^{4N'_f} \bar{\lambda}_J D_\mu \gamma^\mu \lambda_J + m \bar{\lambda}_J \lambda_J \tag{16}$$

where $D_\mu = \partial_\mu + i a_\mu$ and γ^μ are the γ-matrices. After integrating out high energy fermions, we get

$$\mathcal{L} = \sum_{I=1}^{2N_f} \bar{\psi}_I D_\mu \gamma^\mu \psi_I + \frac{\alpha^{-1}}{8\pi} (\boldsymbol{E}^2 - \boldsymbol{B}^2) \tag{17}$$

where the fine structure constant at energy scale E is

$$\alpha^{-1}(E) = \frac{2}{3\pi} [2N_f \ln(E_0/E) + 4N'_f \ln(E_0/m)] \tag{18}$$

where E_0 is the lattice energy scale. We have assumed $m \ll E_0$.

In this paper we propose that light (and other non-Abelian gauge bosons) is originated from the quantum order in our vacuum. To demonstrate this idea, we construct a lattice model with $SU(N_f) \times SU(N'_f)$ spins. We show that in the large N_f and N'_f limit, our lattice model has a ground state characterized by the quantum order Eq. (11). We find that the PSG (or the quantum order) protects the gapless $U(1)$ gauge fluctuations and the massless non-chiral Dirac fermions (when $N_f > 0$). We note that the low energy fermion excitations in our model have the Lorentz invariance, which is also protected by the quantum order. It would be interesting to find a lattice model that gives rise to $U(1) \times SU(2) \times SU(3)$ gauge structure together with *chiral* leptons and quarks.

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